Real Option Valuation vs. DCF Valuation - An application to a North Sea oilfield

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Abstract:

We examine whether the value of an undeveloped oilfield is affected by using real option valuation. Applying a two-factor model dependent on the spot price of Brent and the convenience yield implies a premium over the certainty equivalent method ranging from 20-1000%, for reasonable spot prices. However, the premium over the risk-adjusted method can be negligible since values are dependent on the spot price forecasts of managers. This does not mean that the option criterion should be neglected, considering its implications for the strategic decision of when to optimally invest. The risk-adjusted approach suggests that investment is optimal whenever oil prices surpass $15.69 per barrel, whereas the real option analysis suggests production at prices above $26.72. Moreover, we find evidence of a positive market price of convenience yield risk on the IPE, strongly disagreeing with economic theory.

1. INTRODUCTION

The body of empirical work emphasizing that standard capital budgeting techniques understate the value of projects is growing rapidly. Critics of the DCF criterion argue that cash flow analysis fails to account for the flexibility in business decisions [Triantis and Hodder (1990), Hayes and Abernathy (1980)]. Real option models are more focused on describing uncertainty and in particular the managerial flexibility inherited in many investments. Real options give the firm the opportunity but not the obligation to take action. Such project options typically include the possibility to delay, expand, contract, or liquidate an investment. A company possessing real options is more flexible and thereby more valuable than a company without them. In addition to communicating investment value, these models indicate when to exercise the inherent options in a project.

However, uncertainty is unfortunately often neglected in investment decisions as a result of the complexity in describing it. Copeland, Koller, and Murrin (2000) point out that executives regularly fail to account for the hidden options in projects. Furthermore, managers are claimed to be unfamiliar with practical advances that simplify the understanding and use of real options.

In contrast Butler (2000) emphasizes that companies frequently take decisions that violate the NPV decision rule. He argues that the violations occur as a result of the existing uncertainty in projects. Option pricing theory explains three phenomena: 1) why companies impose higher hurdle rates on investments in foreign countries, 2) why firms often remain in markets where they are loosing money, 3) why firms enter new markets even though the investment obviously has a negative NPV.

Every year petroleum[1] companies bid hundreds of million Pounds for offshore petroleum leases, auctioned out by governments. Performing accurate value estimations is undoubtedly crucial for both governments and the bidding firms. Neglecting the great flexibility in oil ventures could lead to serious undervaluation of assets and miss-allocation of resources in the economy. As a result substantial efforts should be spent in developing valuation models that can describe these opportunities.

This thesis examines whether the value of an undeveloped oilfield differs dependent on if real option valuation or DCF valuation is used. Given that a substantial difference exists, are there any reasons for not using option valuation? Furthermore, are potential hidden values negligible in relation to the information costs incurred by implementing these models? We consider the option to delay investment in an undeveloped North Sea oilfield, where development is being planned. The real option value of the oilfield is estimated by using the Gibson and Schwartz model (1990). Moreover, we extend the model by accounting for the production lag. The valuation is carried out by creating a risk-free portfolio, which implies taking a position in the oilfield and two derivatives. We then use a numerical method to estimate the option value, which is compared to the DCF-value. More precisely, we use an explicit finite difference method to solve the partial differential equation (P.D.E.) satisfying the oilfield value.

The remainder of this thesis is organised as follows. Section 2 outlines oil industry characteristics and the theory of real options. Section 3 contains a review of the literature on real options and petroleum assets. Section 4 presents the option valuation and the DCF model used in our comparison. Section 5 presents our empirical implementation of the models. Section 6 sums up and concludes.

[1] Petroleum is defined as oil and/or gas.

2. THEORY

2.1 Oil Industry Characteristics

Oil is a major resource that continues to be extensively used around the world. However, assuming that petroleum is a homogenous commodity is wrong. It differs depending on where it has been extracted (for instance the Mideast, the North Sea, or the Mexican Gulf), the quality, and to what use it is being refined. Oil extracted from the North Sea is referred to as Brent and is quoted in Dollars per barrel.[2] Futures contracts on Brent are listed on the International Petroleum Exchange in London (IPE) and NYMEX.

Petroleum has very long shelf life as long as it is left for storage in the field. Indeed, if there are no restrictions on the supply of oil and no competition for production, investment can be delayed indefinitely. Improvement in seismic exploration and extraction techniques has opened up production of previously unreachable regions of the sea [Butler (2000)]. As a consequence, crude oil is today being increasingly developed from offshore oil wells, accounting for 35% of the world's oil production [Brandt et al. (1998)].

The owner of an offshore lease must complete three steps before securing petroleum above the ground: Exploration, development, and extraction. Exploration involves considering what area to target in the search of potential oil reserves, which implies taking part in a lease auction.[3] Subsequently, the winning firm will need to complete seismic and drilling procedures to measure how much oil reserves are present in the field, as well as the expenses associated with extracting them. Given favorable results the firm can proceed to the development level. Including the installation of equipment to extract the oil: Typically constructing oilrigs, pipelines, and drilling production wells. Thereby undeveloped reserves are transformed to developed reserves, defined as reserves with productive capacity. After considering whether the surrounding circumstances are prosperous enough the operator can start extracting the petroleum reserves. Consequently, oil investment can be modeled as a compound option, each stage provides an option to complete the next stage [Paddock, Siegel, and Smith (1988)]. Implying that lease valuation requires consideration of all cash flows attributable to the three stages. Figure 1 gives an overview of the options occurring to a leaseholder.

Governments impose restrictions on how long a leaseholder can wait until beginning exploration and development, so called relinquishment requirements. Thereby the exploration and development flexibility can be seen as options occurring to the lease owner. The operator must decide whether to install capacity to extract the developed reserves, or to delay investment. Dixit and Pindyck (1994) argue that the option to delay the development stage is the most valuable in the oil industry. Since it involves the greatest capital expenditures, not easily recovered once investment is undertaken. Exploration and extraction typically involve relatively small expenditures.

A characteristic of the petroleum industry is the fact that several petroleum companies often operate the same platforms and therefore jointly decide when to begin development. Also, the decision to invest can be influenced by the state of the firms other assets. A company can for instance be forced to start development because of poor finances, or by the need to finance exploration activities. Suggesting that delaying investment and beginning development is not always straightforward, as assumed by the real option models.

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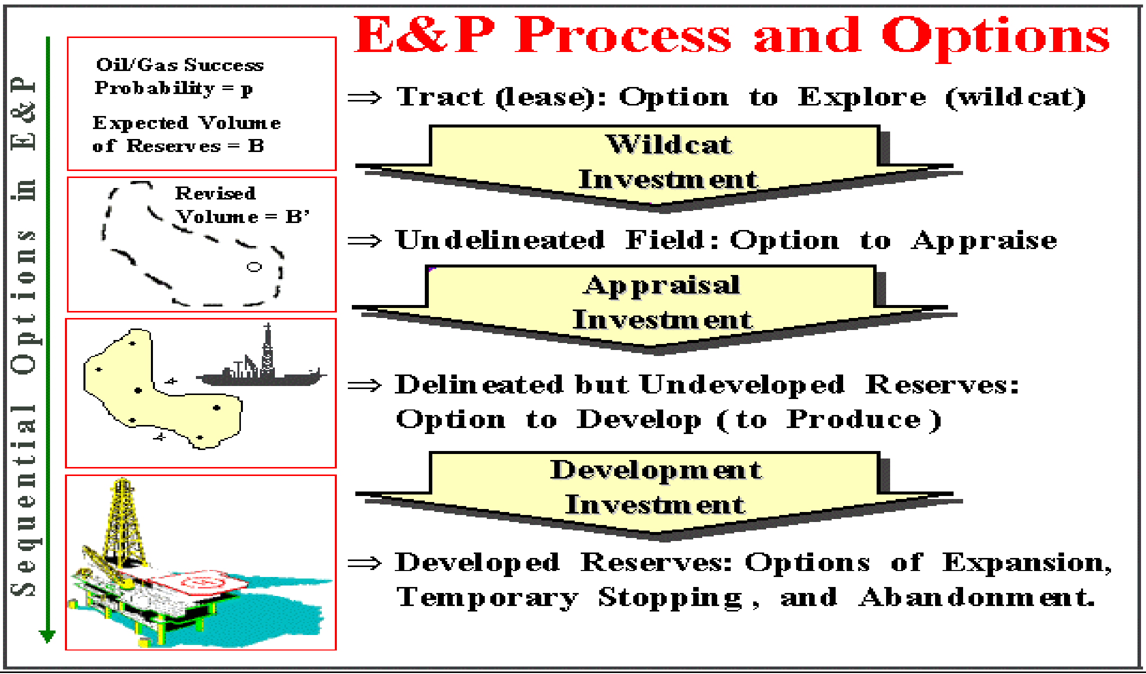
[2] Equivalent to approximately 159 litres.
[3] Typically arranged by a government.

Figure 1
Options Present in Offshore Oil Production

The figure summarizes the many options available to an offshore leaseholder during the exploration and production stages in the petroleum industry. Each stage constitutes an option to complete the next stage.



2.2 Real Option Theory

An irreversible investment cannot be recovered, a sunk cost in other words. Furthermore, irreversible investments frequently involve great uncertainty concerning the future benefits connected to the project. Given that managers inherit some flexibility in deciding when to invest, such a project is always worth more than a similar project without flexibility. Consequently, advocates of real option models argue that the value of a firm can be seen as

$$V_{firm} = NPV + value\ of\ options \quad \textbf{(1)}$$

On the other hand, the defenders of cash flow models claim that the DCF approach has the ability to take into account the options inherent in a project. Of course, this demands that the discount rate changes through project life to reflect the varying risk of future cash flows. While feasible in theory, it is not always achievable in reality.

Flexibility in decision-making includes options to delay, abandon, expand, contract, extend and shorten operations. These options are referred to as real options since they exhibit a claim on real assets. Real option theory can be applied to valuation of natural resources, firms in financial distress, R&D projects, current project expansion or contraction, new product launches, investments in environmental technologies, and the decision to penetrate new markets.

In contrast to option approaches the DCF criterion implicitly assumes that the investment is reversible or if not that the firm has to act now or never. Dixit et al. (1994) point out that many ventures do not meet these conditions; therefore, presence of flexibility in projects should affect the investment decision. Defenders of cash flow methods propose a solution by creating a decision tree and performing NPV calculations at each node, to better capture flexibility. Practitioners refer to the decision tree method as dynamic DCF valuation. Although it constitutes an improvement over the standard DCF method it still fails to incorporate the volatility of the project.

Going through with an irreversible expenditure means foregoing the opportunity to wait for new information, thereby taking on an opportunity cost, which should be included in the investment decision. Indicating that investment is optimal when the value exceeds both the investment expense and the lost option value. Greater uncertainty will increase project value, thereby reducing the actual investment the firm will undertake. The critical spot price S^* is the oil price where the firm should invest, since the option premium at this point is zero. Meaning that the real option value equals the net present value. Dixit et al. (1994) provide evidence that the critical value S^* increases with project volatility. The resemblance to the decision of exercising an American call option is obvious. Exercising an in the money option is not always optimal, since we need to account for the value of waiting before deciding whether to exercise. Table 1 provides an overview of the similarities between real and financial options.

Table 1
Analogy Between Real and Financial Options

The table reports the similarities between the parameters in the Black Scholes formula and a typical real option model. The resemblance makes it easier to understand and implement option methods on real assets.

Black Scholes Financial Options	*Real Options in Petroleum Investment*
Financial Option Value	Value of an Undeveloped Reserve (V)
Current Stock Price	Present Value of Developed Reserve (PV)
Exercise Price of the Option	Investment Cost to Develop the Reserve (k)
Stock Dividend Yield	Net Convenience Yield (δ)
Risk-Free Interest Rate	Risk-Free Interest Rate (r)
Stock Volatility	Volatility of Developed Reserve (σ)
Time to Expiration of the Option	Time to Expiration of Investment Right (τ)

Although many similarities exist between financial and real options there are also some differences

1) In some real option applications it takes time to build the underlying asset, so called production lags.
2) Real options typically have longer time to expiration.
3) Unsystematic uncertainties play an important role in real option models.
4) The issue of when to exercise the option is vital in real options. In contrast, the “Greeks” are insignificant in real option models.

2.2.1 Risk-neutral valuation

Real options along with financial options can be valued under a risk-neutral framework. Constructing a portfolio of the underlying asset and a futures contract assures a certain future payoff. Discounting these certain future cash flows with a risk-free interest rate enables us to find today’s portfolio value. The option value is then calculated by adding the appropriate boundary conditions. Another applicable approach to derive the option value is portfolio replication. More precisely, this implies taking positions in the underlying asset and a risk-free asset that replicates the option payoff, for any price of the underlying asset.

Valuing contingent claims[4] involves substituting the real growth rate of the underlying asset with the risk-neutral growth rate. Reentering the real world we observe two phenomena: The expected growth rate changes and consequently the discount rate changes to reflect the increase in risk. These two events happen to offset each other exactly. Suggesting that the derivative will be valued equivalently in the risk-averse and the risk-neutral world. The advantage of the risk-neutral framework lies in the avoidance of appraising a risk-adjusted discount rate. However, risk-neutral valuation is only appropriate for traded assets. For non-traded assets, affecting the pay off structure of the option, we need to observe the variables real growth rate and the market price of risk.[5] Subsequently, the process can be adjusted to facilitate risk-free discounting.

2.2.2 Stochastic processes

Performing a real option valuation requires a projection of the stochastic process followed by the underlying asset. The process most frequently applied to stocks and commodities is the geometric Brownian motion (GBM). Stating that time and uncertainty are the sole factors affecting the commodity price and supposing that volatility is constant

$$dS = \mu S dt + \sigma S dz \tag{2}$$

where,

μ = expected return on asset S
σ = asset volatility
dz = wiener increment

Eq. (2) states that observed prices are lognormally distributed through time. The variance of dS grows linearly with time and the wiener increment leads to jumpy changes in S. Thus, a wiener process is not differentiable with respect to time. Another way of modeling the evolution of asset prices is to presume that commodity prices are mean reverting [Hull (2000)]. Looking at the Ornstein-Uhlenbeck process in Eq. (3) indicates that the price tends to the long run mean $\overline{P}$. Moreover, the speed of reversion is proportional to the distance between the current position and the equilibrium level. So the variance grows at first but then stabilizes

$$dP = \eta\left(\overline{P} - P\right)dt + \sigma dz \tag{3}$$

where,

η = the speed of reversion to mean $\overline{P}$
P= the current price of the underlying asset

The spot price could also be described by a jump process, given that the commodity has a tendency to be exposed to price shocks. A possible extension to describing jumpy commodity prices is to assume a mean reverting process with temporal jumps. Its strength lies in the strong economic logic, where commodity price volatility is explained as a result of the arrival of exceptional news. Normal news are modeled by mean reversion, whereas a discrete jump

4 Any form of investment right whose value depends on an underlying asset.
5 To be explained in section 2.2.4.

process models the shocks. Despite more realistic assumptions, such a process is unfortunately associated with some problems. Mainly not being able to build a risk-less portfolio and the trouble involved in estimating parameters. Another problem arises in the appraisal of the jump size distribution. All jumps are infrequent so there is a lack of data to estimate these disturbances. A possible solution is to calculate implied parameters from market data.

2.2.3 Convenience yield

Crude oil is primarily held for consumption reasons, not investment. The convenience yield measures the advantages of owning a physical commodity, not obtained by holding a futures contract. Advantages include the possibility of profiting from supply shortages and demand increases. Accordingly, the convenience yield can be seen as an alternative cost for not producing oil and could also be compared to a dividend yield on a stock. Consequently, the commonly used arbitrage argument between futures and spot prices becomes an inequality

$$f \leq Se^{r\tau} \qquad \textbf{(4)}$$

where,

τ = time to maturity (T-t)
r = risk-free interest rate
S = spot rate

Benefits of holding Brent are clearly higher when there is a low availability of crude oil in the world market. Conversely, the higher inventories the lower convenience yield. In view of that, the convenience yield can be argued to describe investor's forecasts regarding the future supply of the commodity [Hull (2000)]. In addition, the convenience yield of a commodity can be different for various users and can vary overtime. When inventories of a commodity decrease, the spot price should increase, as should the convenience yield. All else equal, increases in the convenience yield will lead to lower futures prices for long-term maturity contracts. Most models of stochastic processes use a net convenience yield, the convenience yield less the storage costs, to describe futures prices

$$f = Se^{(r-\delta)\tau} \qquad \textbf{(5)}$$

where,

δ = net convenience yield

Bearing in mind the notion of the convenience yield as an alternative cost suggests that the value of delaying investment decreases as the benefit of holding crude oil increases. On the contrary, if the convenience yield decreases or becomes negative the value of the option to delay investment increases. Although negative convenience yields do not have a rational economic interpretation. Either benefits from holding oil inventories exist or they do not. The main disadvantage from holding oil reserves is the storage cost, but these costs are already included in the net convenience yield.[6]

2.2.4 Market price of risk

Any contingent claim on an asset, traded or not, can be priced in a world with systematic risk by subtracting the corresponding risk premium in market equilibrium and then behaving as if the world is risk-neutral. In the risk-neutral world the market price of risk is zero. It is independent of the nature of the derivative and is defined as

$$\lambda = \frac{(\mu - r)}{\sigma} \tag{6}$$

where,

μ = expected return on the asset
$\mu = g + \delta$, (net convenience yield δ, and the expected growth rate in price g)[7]
σ = standard deviation of the state variable
r =risk-free interest rate

Eq. (6) indicates that λ is equivalent to the sharp ratio of a stock index. Multiplying the amount of risk σ, with the market price of risk λ, results in the risk premium for the underlying asset. If the risk premium is positive investors require a higher return to compensate for the state variables risk. Conversely a negative premium causes investors to require a lower return. Implying that the state variable has the effect of reducing rather than increasing systematic risks in portfolios. Eq. (6) is valid for both consumption and investment assets, since we define expected return as the growth rate in the spot rate plus the convenience yield.[8]

Pricing a contingent claim on any underlying asset requires the estimation of the growth rate in a risk-free world. The correct growth rate is found by subtracting the risk premium of the stochastic variables. Eq. (6) verifies that the risk-free growth rate of an investment asset is *r*, whereas the risk-free growth rate of a consumption asset is ($r - \delta$). Confirming the fact that no risk premiums need to be estimated for traded state variables, in order to value derivatives in a risk-free world. In contrast, dealing with non-traded assets[9] demands estimation of the exogenous risk premium and risk-adjusted growth rate.

3. PREVIOUS LITERATURE

3.1 General Studies on Real Options

Brennan and Schwartz (1985) develop a one-factor model for evaluating natural resource investments. The spot price is assumed to be the only state variable, following a geometric Brownian motion. Considering a hypothetical Copper mine they display spot prices at which it is optimal to abandon, temporarily close, or delay operations. Wiklund and Ösund (1997)

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[6] In theory though, the net convenience yield can be negative, in case storage costs are perceived to be larger than benefits.
[7] The similarities with the expected return on a stock are obvious.
[8] The convenience yield of an investment asset is zero and is replaced by a dividend yield.
[9] Implying that the risk in these parameters cannot be hedged.

use the same model to value a lead-zinc ore mine in Sweden. In addition, they add an exchange rate process to describe the option value outside the United States. The study indicates an option premium of 10% for current metals prices. For reasonable spot rates the premium varies around 5-50%, for lower rates it amounts to more than double the DCF value. Moel and Tufano (2002) carry out an empirical study using the Brennan and Schwartz model to describe the option to open and close a developed gold mine. Focusing on 285 North American mines, findings indicate that the real option model is successful in describing the opening and closing decisions of firms, for the period 1988-1997. Furthermore, they compare the option model to a model not accounting for volatility. The former is determined to be significantly better as a predictor, implying that volatility improves predictive power. Also, the decision to shut down a mine is connected to firm-specific managerial factors, not considered in an ordinary real options model. Another interesting finding suggests that abandonment decisions are influenced by the fortunes of the firms other mines. Coordination between different operating partners however, does not influence the likelihood that a mine will remain open.

McDonald and Siegel (1986) examine the option to invest in a project. Both the present value of benefits and investment costs are modeled as stochastic following a geometric Brownian motion. Although the underlying assumption states that project life is infinite, project cash flows are allowed to jump to zero by adding a Poisson jump to the process. For reasonable parameters the option premium of a project is proven to be significant and ranges from 10 to 30%.

Ingersoll and Ross (1992) point out that the presence of interest rate risk suggests that all projects possess flexibility value, in an uncertain economic environment. They argue that the effect of uncertainty on investment delay is sizeable. Nevertheless, the option to delay can be negligible sometimes. Especially in new fast growing businesses, where it is necessary to be first in capturing the growth. Implying that postponing investment not only postpones receipt of each cash flow but also looses a period's growth.

Whether option valuation is applied or not is essential to understanding investment decisions made in the economy. Lindblom and Turgay (2000) interview several investment companies in Stockholm, questioning whether corporations use option-pricing techniques. Findings indicate that most firms do not, since complex models require estimation of volatility and explanation of the model to clients.

Linden, Lindskog, and Plemic (2001) examine how Internet companies are valued in practice, based on twenty interviews with practitioners in Stockholm, London, New York, and Boston. Answers indicate that DCF valuation and multiples are the most commonly used valuation techniques in practice. Moreover, the usage of option approaches is limited, since the knowledge of the approach is still in its infancy. The technique is often regarded as complex and time-consuming. There also seems to exist consensus among analysts that the DCF method will continue to be the dominant tool in the future, with option valuation as a complement. Finally, the authors conclude that option theory is utilized more as a strategic tool.

3.2 Studies on Petroleum Pricing

Paddock et al. (1988) value 21 selected offshore petroleum leases in the Mexican Gulf by using a one-factor model, where uncertainty is generated by the spot price of oil following a geometric Brownian motion. Results indicate that historic government valuations have tended to underestimate industry bids. Using the one-factor model enables the authors to present values closer to industry bids.[10] The option premium ranges from 10-50% calculated as a mean of 21 leases. But the highest bids were typically more than double the option value. A plausible explanation could be that they disregard the abandonment option. Results also suggest that reserves with low investment costs are likely to be developed earlier.

Hurn and Wright (1994) examine the influence of economic variables on the lag between field discovery and field development, using data from 108 North Sea oilfields. The data indicate that the expected price of oil and the level of reserves are important in influencing the appraisal duration, but the variance of the oil price is not. Furthermore, non-economic features of the fields significantly influence the appraisal lag. Hence, findings do not provide strong support for real option models. Geology, not economics appears to drive the production start up duration, even more than in the appraisal duration.

Smith and Mccardle (1996) value oil properties by using option pricing methods to value risks that can be hedged and decision analysis to value risks that can not be hedged. The valuation method is developed in a discrete-time, finite-horizon framework where uncertainties are gradually resolved. The authors illustrate the benefits of their valuation methodology by comparing the results to previous results given by conventional methods. Although conventional methods can determine the correct value they do not take risk aversion and market opportunities into account.

Laughton (1998a) discusses the growing worry in the petroleum industry concerning the DCF method as a valuation tool. Focusing mainly on the underestimation of reserve value, the bias towards building too much production capacity, which leads too inefficiency, the lack of consideration for the flexibility in the project, and the lack of ability to evaluate the unique risk profiles of similar projects. He argues that instead of adjusting the DCF valuation by for instance using a higher discount rate it would be better to avoid the biases in the first place.

Laughton (1998b) examines how project value depends on price and reserve size uncertainty in the offshore petroleum industry. The considered option is an abandonment option. His findings indicate that both types of risk increase project value. Furthermore price uncertainty leads to delay of all actions, while greater reserve size risk leads to sooner action. By using two different permanent shock price models he estimates the option premium to be over 40% for reasonable parameters.

Gibson et al. (1990) develop a two state variable model for valuation of financial and real oil assets, dependent on the spot price and instantaneous convenience yield. The model is shown to perform well in pricing short-term futures contracts, but the performance worsens for contracts of more than six months maturity. Nonetheless, using weekly updates of the market price of risk enhances the pricing accuracy. Spot prices seem to follow a random walk while the convenience yield is found to be strongly mean reverting. Schwartz (1997a) compares three different models taking mean reversion in crude oil prices into account. He tests their ability to price futures contracts and the implications for valuation of real assets. The first model is a simple one-factor model in which the logarithm of the commodity price is assumed to follow a mean reverting process. The second model is the Gibson and Schwartz model and the third extends the former model by including stochastic interest rates. All three models are compared with the DCF method and a real option model neglecting mean reversion, i.e. a geometric Brownian motion. Results suggest that pure mean reversion models are incapable of predicting the term structure of oil futures contracts. Conversely, valuations using the multi-factor models are remarkably accurate. Pricing contracts with maturities of up to 18 months indicates that the term structure prediction is indistinguishable for the two models. Although predicted prices can vary considerably for longer maturities. For maturities of up to ten years the three-factor model performs slightly better. On the other hand, model

[10] Valuing both the exploration and development stage.

three is very sensitive to the assumption made concerning the long run discount yield. For in the money projects model three gives the highest value, followed by model two, the GBM model, and last the simple mean reversion model. The option premiums of model two and three over the DCF method typically range around 20-60% for reasonable spot prices.

Schwartz (1997b) develops a one-factor model, with practically the same implications as the more advanced two-factor model, when applied to valuing long-term commodity assets. The single-factor model is derived from the two-factor model, but is easier to use when valuing complex multiple real option projects. It constitutes a good approximation in the long run and has to be solved numerically. The difference lies in the assumption of a constant convenience yield, which is a function of the other parameters.

Cortazar and Schwartz (1998) implement the Gibson and Schwartz model to value an undeveloped oilfield and to determine the optimal time to invest using a Monte Carlo simulation method. The advantage of using Monte Carlo simulation lies in the possibility to increase the number of state variables considered in the analysis.

Schwartz and Smith (2000) propose a new two-factor model, allowing for short-term mean-reverting variations in prices and some uncertainty in the equilibrium level to which the prices revert. Assumptions in respect to the convenience yield are not made, but the model is equivalent to the classic two-factor model. Short-term price deviations are modeled as a linear function of the instantaneous convenience yield. The authors argue that this model should be easier to apply, since it demands estimation of one parameter less than the ordinary two-factor model.

3.3 Evidence of Mean Reversion in Oil Prices

Laughton and Jacoby (1993) show that failure to accommodate for mean reversion in oil prices leads to overestimation of risk and systematic biases in capital budgeting decisions. Dixit et al. (1994) sum up the previous research on mean reversion in crude oil spot prices, by pointing to the fact that weak mean reversion can be seen over a thirty year period, but is nonexistent for the past hundred years.

Bessembinder, Coughnenour, Seguin, and Smoller (1995) provide further evidence on mean reversion in asset prices. They use the term structure of futures prices to test whether investors anticipate mean reversion in spot prices. An inverse relationship between prices and the futures term slope constitutes evidence that investors expect mean reversion in spot prices. Their approach suggests that mean reversion occurs for two different reasons. Attributable to either positive correlation between spot prices and the convenience yield, or a negative relationship between interest rates and spot prices. Eq. (5) displays the possible reasons for mean reversion. For crude oil the results indicate that mean reversion arises solely from positive co-movement between prices and the convenience yield. Their conclusion reinforces the study by Gibson et al. (1990), suggesting strong mean reversion in the convenience yield.

Schwartz (1997a) study also reveals strong mean reversion in the convenience yield for oil. He concludes by saying that DCF induces investment to early, but the real options approach induces investment to late when it neglects mean reversion.

3.4 Summary of Previous Literature

The vast majority of real option studies model the spot price process as a geometric Brownian motion. Option premiums typically vary around 10-50% for reasonable parameters. Moreover, the project value at which investment is optimal is typically more than double the investment cost for many projects. For option valuation of undeveloped petroleum reserves the premium typically ranges between 20-60%, for all spot price models. Although not unanimous, some research indicates that the lag between discovery and production is dependent on the geological features of the field, rather than spot rate variability. Contrary evidence is presented in a recent study on gold mines, suggesting that spot price volatility has explanatory power.

Empirical research provides strong evidence of the excellent pricing ability of multi-factor models in valuing oil futures contracts. Models assuming mean reversion or a random walk in the spot rate clearly underperform. While models presuming that uncertainty is inherent from several sources, notably the spot rate, the net convenience yield, and interest rates do excellently in pricing futures. Spot prices follow a random walk in all these multi-factor models and the convenience yield is mean reverting. The two-factor model performs slightly better in the short run and the three-factor model somewhat better in the long run.

Most studies indicate that the knowledge and use of real option approaches is limited. Practitioners primarily use it as a complement to DCF analysis and the applied option models are typically uncomplicated binomial trees.

4. METHODOLOGY AND DATA

4.1 DCF Valuation

Applying cash flow valuation to evaluate oilfield projects constitutes an easy and quick way to gather information for investment decisions. Difficulties in carrying out the valuation are more likely to occur from oil reserve uncertainty, than from the model itself. The DCF approach implicitly assumes that the investment is undertaken immediately and that future cash flows are predictable. Furthermore, these cash flows are supposed to be exposed to a constant systematic risk, reflected by a constant risk-adjusted discount rate. Oil price volatility is not explicitly accounted for in the DCF valuation, although it can be argued that systematic price risk is considered in the discount rate.

In this particular case all geological conditions of the undeveloped oilfield are known, so once the investment has been committed the yearly oil production and annual costs are presumed to be certain. All exploration and appraisal expenses have been committed and are therefore treated as sunk costs. The relinquishment requirement expires in January 2012. Meaning that the oilfield operator can delay investment prior to this day. However as previously emphasized only the real option approach accounts for the possibility to delay investment, since the DCF criterion assumes immediate investment. Consequently, the relinquishment requirement does not affect the cash flow valuation. We do not account for the quality of the oil reserves; rather we assume that all reserves can be sold at market prices. We regard this to be a plausible assumption for our oilfield. The forecasted oil price for the entire production period is set at OPEC's long run price target of $20 per barrel and more than half of the recoverable oil is extracted in the first two years of production. All the natural gas present in the offshore field will not be considered in the valuation, since it constitutes a minor part of project value.[11]

The DCF valuation assumes that the decision to invest is taken on Jan.1, 2001. Since there is a three-year production lag cash flows are expected at the beginning of year 2004. Crude oil is then extracted for nine more years until early 2013, when production is abolished. All annual costs are multiplied each year with the Dollar/Pound forward exchange rate. This adjustment is necessary taking into consideration that project costs are in British Pounds and revenues are received in U.S. Dollars. We calculate the forward rate each year by using the

[11] A predicted price of $1.80 per thousand square cubic feet (scf) of gas indicates a gas reserve value of $21 million, clearly negligible in relation to the amount of recoverable oil reserves.

ten-year U.K. and U.S. government bonds at the end of December 2000. No depreciation is made and we disregard any changes in operating working capital. After-tax cash flows are discounted with a calculated WACC[12] reported in Table 2 along with the rest of our oilfield data. See Appendix C for a detailed overview of the DCF valuation

$$NPV = -k + \sum_{t=T}^{N} e^{-WACC*t}\left[(Q_t S_t - C_t X_t)(1-h_t) + h_t D_t\right] \quad \textbf{(7)}$$

where,

k = initial investment
N = life of the oilfield once production has begun
WACC = risk-adjusted continuous discount rate
Q_t = number of barrels of oil to be extracted in year t
S_t = assumed Brent spot price
C_t = total cost of production year t
X_t = forward exchange rate year t
h_t = corporate tax rate
D_t = planned depreciation year t

Table 2
Oilfield Characteristics and Financial Parameters[13]

The table reports oilfield characteristics and financial parameters used in the DCF valuation. As emphasized previously the volume of natural gas does not enter into the valuation. Reuters have provided us with the interest- and exchange rate data.

Parameter	*Data*
Volume of oil initially in place	113 MMstb[14]
Volume of oil technically recoverable	19.5 MMstb
Volume of natural gas	11.5 Bscf [15]
Relinquishment requirement[16]	11 years
Production lag	3 years
Assumed debt ratio	0.70
Estimated Beta	0.70
Corporate tax rate[17]	15%
Assumed risk premium	5.00%
U.S. ten-year government bond 2001-01-01	5.00%
Assumed default premium	2.00%
Cost of debt	7.00%
Resulting WACC	6.72%
Assumed Brent price/barrels in 2004	$20
Forward Exchange rate 2004-01-01 ($/£)	1.483

[12] The WACC has been determined using estimates of the debt ratio, default premium, beta and an assumed risk premium to calculate the cost of equity.
[13] Due to confidentiality we are unable to reveal the name of the oilfield and the company providing the data.
[14] Means millions of stock tank barrels.
[15] Means billions of square cubic feet.
[16] Used only in the real option valuation.
[17] An approximation of the annual tax rate given to us by the oilfield operator.

4.2 Real Option Valuation

Carrying out a real option valuation implies making assumptions about which state variables effect the value of the contingent claim. Furthermore, the evolution over time of these variables needs to be modeled with stochastic processes. Time series data should be monitored closely to determine a suitable process for each variable. The importance of observing the evolution of stochastic variables lies in the fact that their predicted pattern affects the derivative value.

When it comes to crude oil previous research has shown that one-factor models, predicting that spot prices follow mean reverting or geometric Brownian motions, perform poorly in valuing futures contracts [Schwartz (1997a)]. The success of the two- and three-factor models suggests that taking convenience yield variability into account is necessary in order to achieve accurate pricing of futures and forward contracts.

We have therefore decided to use the two-factor model developed by Gibson et al. (1990) and tested by Schwartz (1997a). All recent studies document the two-factor models remarkable performance in pricing contracts. Although performing slightly worse in valuing long-term contracts it involves a less complicated numerical solution than the three-factor model. The underlying assumptions state that the spot price and the convenience yield vary stochastically over time.[18] Spot prices are imagined to follow a geometric Brownian motion with drift and the convenience yield follows a mean reverting Ornstein-Uhlenbeck process

$$dS = (\mu - \delta)Sdt \;\; + \sigma_S S dz_S \qquad \textbf{(8)}$$

$$d\delta = \eta(\alpha - \delta)dt \;\; + \sigma_\delta dz_\delta \qquad \textbf{(9)}$$

$$dz_S dz_\delta = \rho \; dt$$

where,

S = spot rate
μ = expected return from holding crude oil
δ = annualized convenience yield
η = annualized reversion rate in the convenience yield
α = mean convenience yield
dz_s and dz_δ = correlated wiener increments
ρ = correlation between wiener increments

Eq. (8) and (9) describe the evolution of the two state variables in the real world. The drift term of the spot rate is adjusted by the convenience yield since we have defined μ as the total expected return. Consequently, (μ-δ) translates into the growth rate of the spot price. Unfortunately stochastic processes are not differentiable, however, by using Ito's Lemma we can define the instantaneous value change of any contingent claim. Given that the price $f(S,\delta,\tau)$ of a crude oil derivative is a function of two variables, Ito's generalized Lemma[19] helps us determine the instantaneous price change.

[18] So the volatility in the project is modeled purely by spot price and convenience yield uncertainty. In theory project volatility could be affected by other factors. However, since we treat the amount of recoverable reserves as a known amount it is fair to assume that the volatility of the project is determined by the spot price and the convenience yield.
[19] See appendix A.

$$df = \left[\frac{\partial f}{\partial S}(\mu - \delta)S + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta)) - \frac{\partial f}{\partial \tau} + \frac{1}{2}\left(\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 + 2\frac{\partial^2 f}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\ S \right) \right] dt + $$
$$+ \frac{\partial f}{\partial S}\sigma_S S dz_S + \frac{\partial f}{\partial \delta}\sigma_\delta dz_\delta \qquad \textbf{(10)}$$

Eq. (10) describes the instantaneous price change of any oil contingent claim dependent on S and δ. Using the above expression enables us to create a risk-free portfolio. More specifically by taking a position in two derivatives and a position in crude oil (purchasing oil reserves) we create a risk-neutral portfolio leading to the differential equation below[20]

$$\frac{\partial f}{\partial S}(r - \delta)S + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta) - \lambda \sigma_\delta) + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{1}{2}\frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 + \frac{\partial^2 f}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\ S - \frac{\partial f}{\partial \tau} - rf = 0 \qquad \textbf{(11)}$$

where,

λ = market price of convenience yield risk
r = risk-free interest rate

The price of any oil contingent claim must satisfy the above partial differential equation. Eq. (11) indicates that determining the value of a contingent claim demands estimation of the market price of convenience yield risk, since it represents a non-traded asset. To value a specific derivative, for instance a call option, we just add the appropriate boundary conditions. The price of a futures contract F(S,δ,T) is given by

$$\frac{\partial F}{\partial S}(r - \delta)S + \frac{\partial F}{\partial \delta}(\eta(\alpha - \delta) - \lambda \sigma_\delta) + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma_s^2 S^2 + \frac{1}{2}\frac{\partial^2 F}{\partial \delta^2}\sigma_\delta^2 + \frac{\partial^2 F}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\ S - \frac{\partial F}{\partial \tau} = 0 \qquad \textbf{(12)}$$

subject to the initial condition

$$\text{F(S,δ,0)} = \text{S} \qquad \textbf{(13)}$$

Jamshidian and Fein (1990) have shown that the analytical solution to Eq. (12) is

$$\text{F}(\text{S}, \delta, T\ ) = S * \exp\left[-\delta\ \frac{1 - e^{-\eta T}}{\eta} + A(T) \right] \qquad \textbf{(14)}$$

where,

$$\text{A(T)} = \left(\text{r} - \hat{\alpha} + \frac{1}{2}\frac{\sigma_\delta^2}{\eta^2} - \frac{\sigma_S \sigma_\delta \rho}{\eta} \right) T + \frac{1}{4}\sigma_\delta^2 \frac{1 - e^{-2\eta T}}{\eta^3} + \left(\hat{\alpha}\ \eta + \sigma_S \sigma_\delta \rho - \frac{\sigma_\delta^2}{\eta} \right) \frac{1 - e^{-\eta T}}{\eta^2} \qquad \textbf{(15)}$$

$$\hat{\alpha} = \alpha - \frac{\lambda \sigma_\delta}{\eta}$$

[20] Deriving the partial differential equation by creating a risk-free portfolio is complicated. So we derive it using a two-factor bond pricing technique instead, summarized in Appendix A.

Having an analytical expression for valuing futures enables us to compute the term structure of Brent contracts for any maturity. That is on any given day a spot price S and an annual convenience yield δ, implies a certain term structure of futures prices. Accordingly, we calculate futures prices for 12 years[21] by using Eq. (14) for the term structure on Jan.1, 2001. Subsequently, the project is evaluated as if the decision to invest is already taken. The present value of cash flows from the oilfield is obtained through a certainty equivalent (CEQ) approach. More specifically, the computed term structure of futures for twelve years is used to estimate cash flows, which are discounted at the risk-free interest rate.[22] We assume that all production will take place according to plan once development of the field starts and that annual depreciation is tax deductible.[23] The CEQ value of the developed project is then

$$NPV_{CEQ} = -k + \sum_{t=T}^{N} e^{-r\,t}\left[(Q_t F(S,\delta,T) - C_t X_t)(1-h_t) + h_t D_t\right] \quad \textbf{(16)}$$

$$PV_{CEQ} = \sum_{t=T}^{N} e^{-r\,t}\left[(Q_t F(S,\delta,T) - C_t X_t)(1-h_t) + h_t D_t\right] \quad \textbf{(17)}$$

k = initial investment
N = life of the oilfield once production has begun
r = yield on a ten-year government bond
Q_t = number of barrels of oil to be extracted in year t
C_t = total cost of production year t
X_t = forward exchange rate year t
h_t = corporate tax rate year t
D_t = planned depreciation year t

To value the option to invest[24] we need an expression for the value change of our project. Taking a position in two derivatives and the oilfield, which means adding Eq. (16) to (11), derives the P.D.E. satisfied by the value of the option to invest V(S,δ,τ) [Schwartz 1997b]

$$\frac{\partial V}{\partial S}(r-\delta)S + \frac{\partial V}{\partial \delta}(\eta(\alpha-\delta) - \lambda\sigma_\delta) + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma_S^2 S^2 + \frac{1}{2}\frac{\partial^2 V}{\partial \delta^2}\sigma_\delta^2 + \frac{\partial^2 V}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\, S - \frac{\partial V}{\partial \tau} - rV$$

$$-k + \sum_{t=T}^{N} e^{-r\,t}\left[(Q_t F(S,\delta,T) - C_t X_t)(1-h_t) + h_t D_t\right] = 0 \quad \textbf{(18)}$$

Subject to the terminal condition

$$V(S,\delta,0) = \max\,[\mathrm{NPV},0] \quad \textbf{(19)}$$

and the lower and upper boundary conditions

$$V(0,\delta,\tau) = 0 \quad \textbf{(20)}$$

[21] Remember that there is a three-year production lag.
[22] All costs are assumed to be predictable as mentioned previously.
[23] In the case of our oilfield no depreciation is made, $D_t = 0$.
[24] Equivalent to the value of the oilfield.

$V(S,\delta,\tau)/\ PV \rightarrow 1,\ \text{as } PV \rightarrow \infty.$ **(21)**

$V(S,\delta,\tau) \rightarrow PV \text{ as } \delta \rightarrow -\infty$ **(22)**

$V(S,\delta,\tau) = 0 \text{ as } \delta \rightarrow \infty$ **(23)**

The above lower and upper boundary conditions state that the option to delay is worthless if the spot rate is zero or if benefits of owning crude oil become infinitely large. While the other boundary conditions proclaim that if the present value of future cash flows goes to infinity or the convenience yield becomes infinitely small, the value of the option will equal the CEQ value of the project.

There does not exist a closed form analytical solution to the P.D.E. (18), describing the option to invest. Therefore the P.D.E. is solved numerically subject to the above five boundary conditions, for our estimated parameters. The numerical method used is an explicit finite difference method.[25] Since the option to delay is an American option we iterate backwards at each discrete point in time, to investigate whether investment is optimal, for all possible values of the convenience yield and the Brent spot rate.

We wish to emphasize that two different DCF valuations are carried out. A certainty equivalent estimation, used as input in the option valuation, and a risk-adjusted method described in the previous section. Thereby the option premium is better pictured, considering that professionals regularly use risk-adjusted methods. The difference lies in the forecasted oil price and the used discount rate, all other parameters are subject to the same assumptions. Obviously these two techniques can imply quite different oilfield values. Certainty equivalent values can never be higher than the corresponding real option value, according to the terminal condition in Eq. (19). However, the risk-adjusted valuation can imply either a higher or lower value than the option value, considering that it has not been developed in the same risk-free framework.

4.3 Estimation of Real Option Parameters

Since the undeveloped oilfield will provide the leaseholder with cash flows over a decade long period, all model parameters shall be estimated over a historic period of the same length.

4.3.1 Estimation of the convenience yield

Evolution through time of non-traded assets like the convenience yield can be estimated using market data. Applying the well-known arbitrage relationship between futures and spot prices of commodities, $F = Se^{(r-\delta)(T-t)}$, enables the calculation of the convenience yield.[26] The Brent spot price is not observable[27] and is therefore approximated by the closest maturity futures contract traded on the IPE.[28] The futures contracts used are the two nearest maturity contracts throughout our ten-year estimation period. We use a three-month U.S. T-bill with the closest maturity to the futures contracts as a proxy for the interest rate. Rearranging Eq. (5) yields Eq. (24),[29] allowing us to determine the implied annualized convenience yield each week using futures data over the period Jan.1990-Dec.1999.

[25] The numerical solution was programmed in Matlab. We would like to thank Daniel Appelö, NADA KTH, Sweden, for helping us programming the numerical solution.
[26] Assuming a constant convenience yield and no interest rate uncertainty.
[27] Producing companies trade most of the volume on a spot basis, with virtually no formal term contracts.
[28] There are 17 monthly Brent futures contracts available on the International Petroleum Exchange.
[29] T=1/12 since we use the two nearest contracts.

$$\delta_{T-1,T} = r_{T-1,T} - 12\ln\left[\frac{F(S,T)}{F(S,T-1)}\right] \tag{24}$$

where,

$\delta_{T-1,T}$ = *annualized one-month convenience yield*
$r_{T-1,T}$ = *corresponding risk-free T-bill rate*
$F(S,T-1)$ = *spot price approximated by the closest maturity futures contract*

4.3.2 Appraising joint stochastic process parameters

The advantage of holding crude oil is an increasing function of the spot price. Bearing in mind the strong relationship between the Brent price and the convenience yield accentuates the need to consider that the spot price and the convenience yield have correlated residuals. Hence, it is important to use an effective method when measuring parameters. Consequently, parameters η, α, and σ_δ are estimated by performing a seemingly unrelated regression (S.U.R.), following Gibson et al. (1990). A seemingly unrelated regression considers the correlation between residuals of two regressions to achieve more precise estimates. Provided that there is no correlation between the residuals of Eq. (25) and (26) the S.U.R. will yield the same result as an OLS regression. Otherwise estimated parameters will differ for the two regression techniques. Weekly time series data of the state variables are run simultaneously against their lagged values, to estimate the coefficients β_0, β_1, and b between Jan.1990-Dec.1999. Eq. (25) is the linear discretized approximation of Eq. (9) namely

$$\Delta\delta_t = \beta_0 + \beta_1\,\delta_{t-1} + e_t \tag{25}$$

$$\ln(S_t / S_{t-1}) = a + b\ln(S_{t-1} / S_{t-2}) + \varepsilon_t \tag{26}$$

Implying that the evolution of the convenience yield through time can be considered as observations of the linear relationship between $\Delta\delta_t$ and δ_{t-1} in the presence of the noise, e_t, which has the distribution ~N(0,1). According to Dixit et al. (1994) the long run mean of the convenience yield α and the annualized reversion rate, η, is given by

$$\alpha = -(\beta_0 / \beta_1) \tag{27}$$

$$\eta = -\ln(1+\beta_1)/\Delta t \tag{28}$$

Moreover, they demonstrate that the standard deviation of an Ornstein-Uhlenbeck process is defined by

$$\sigma = \sigma_e \sqrt{\frac{\ln(1+\beta_1)}{(1+\beta_1)^2 - 1}} \tag{29}$$

where,

σ_e = *standard error of* β_1 *in the regression*

Studying time series data for the ten-year estimation period generates the standard deviation of the Brent spot price and the correlation between the two state variables. Accordingly, there is only one remaining parameter to appraise, the exogenous market price of convenience yield risk, λ.

4.3.3 Estimation of market price of convenience yield risk

Estimation of the exogenous market price of risk, λ, is performed correspondingly. We use prices of Brent futures contracts traded on the IPE and compare market prices to our own estimates generated by the valuation model. Minimizing the sum of squared errors (SSE) for the chosen contracts by varying λ derives the optimal value of the market price of risk. More specifically, we use the analytical solution in Eq. (14) to the P.D.E. (12) subject to the corresponding boundary condition Eq. (13). For a total of 1566 weekly futures observations, the corresponding annualized convenience yield and T-bill rate are used to compute theoretical prices for each week. Since focus should be on the long run market price of risk we estimate it using weekly futures prices over the ten-year period. We only intend to use contracts traded under fair liquidity, to ascertain that the contracts are fairly priced. Therefore contracts with daily volume consistently below 300 contracts during the estimation period are not considered. Furthermore, there was practically no trading of longer maturity contracts in the beginning of the estimation period Jan.1990-Dec.1999. As a result we are only able to use futures with maturity of two to four months.

Research by Gibson et al. (1990) indicates that the two-factor model's assumption of a constant market price of risk is unsatisfactory, meaning that it is a highly volatile parameter. Nevertheless, precise contract valuation can still be achieved despite the unrealistic assumption. So to verify whether our optimized λ is sensible, we test the out of sample pricing ability[30] by pricing futures between Jan.2000-Jul.2000. All longer maturity futures with reasonable liquidity are included. Accordingly, we use out of sample contracts with maturities between two to eight months. For each contract the mean pricing error (MPE) and the root mean square error (RMSE) are computed, enabling us to determine whether there exists a general overpricing and the magnitude of it. The errors implied by our parameter estimates are then compared to the residuals of Gibson et al. (1990) and Schwartz (1997a). Given that the performance of our analysis for shorter contracts matches the former studies, we presume that the relative performance should not be worse for long-term contracts.

Unlike Schwartz (1997a), we were unable to obtain any long-term oil forward contracts. Thus, we do not estimate parameters from long-run forwards and are therefore unable to test our short-run parameter estimates on the valuation of long-run contracts.

5. EMPIRICAL ANALYSIS

5.1 Real Option Parameters

Performing a reliable real option valuation requires controlling the distribution assumptions of the state variables. To examine whether the random walk and lognormal distribution of Brent prices is supported by the data we analyze the time series of our approximation for the spot price, according to Eq. (26). The logarithms of the price ratios are regressed against their lagged values, results are reported in Table 3.

[30] Again using Eq. (14).

Table 3
Time Series Properties of ln(St/St-1)

The table reports the regression parameters using weekly price notations between Jan.1990-Dec.1999. Parameters are also reported for two subperiods to detect any variation through time.

Period	*b*	*t(b)*	*DW*[a]	R^2	σ_S[b]	*N*[c]
Jan. 90- Dec. 99	-0.1047	-2.41	1.98	0.01	33.66%	520
Jan. 90- Dec. 94	-0.1396	-2.27	1.96	0.02	35.51%	258
Jan. 95- Dec. 99	-0.0631	-1.01	1.99	0.00	31.94%	258

OLS model: $\ln(S_t/S_{t-1}) = a + b\ln(S_{t-1}/S_{t-2}) + \varepsilon_t$

[a] DW denotes the Durbin-Watson statistic.
[b] σ_S denotes the annualized standard deviation of $\ln(S_{t-1}/S_{t-2})$ over the period.
[c] N denotes the number of observations.

Table 3 indicates that the notion of spot prices following a random walk is not fully supported by the regression parameters. Nonetheless, dividing our sample into two estimation periods suggests that the drift term is not significant for the entire period. Studying the explanatory power of the regression suggests that historic returns are unable to explain future returns. Observing Fig. 2 provides no evidence contradicting the random walk assumption and there seems to be no indication of strong mean reversion in spot prices. So we draw the conclusion that the significance of the drift term is just noise. The volatility of the spot price is fairly constant over the entire sample. Supporting the underlying assumption of constant spot price volatility in the real option framework.

Figure 2
The Spot Price of Brent in Dollars

The below figure displays the evolution of the Brent spot price, approximated by the nearest maturity contract on the International Petroleum Exchange, between Jan.1990-Dec.2000.

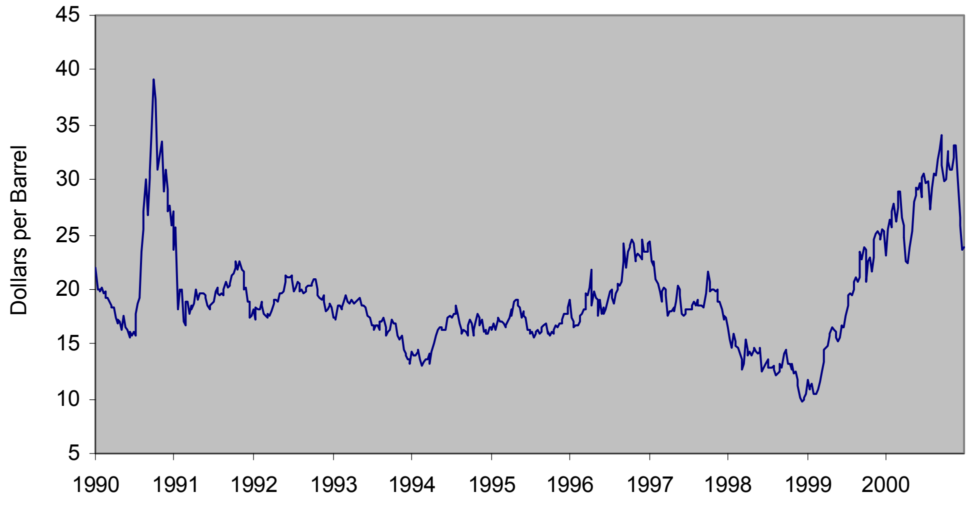


Table 4
Estimation of the Parameters of the Joint Stochastic Process[a]
Followed by $\Delta\delta$ and $\ln(S_t/S_{t-1})$

The table reports the parameters of the seemingly unrelated regression performed using weekly price notations between Jan.1990-Dec.1999. Parameters are also reported for two subperiods to detect any variation through time. The logarithmic spot price change and the change in the convenience yield were regressed jointly against their lagged values.

Period	η^b	$t(\eta)$	α	$t(\alpha)$	$\sigma_\delta{}^b$	$\rho_{S,\delta}$	DW^c	R^2	N^d
Jan. 90-Dec. 99	5.2988	5.58	0.1183	2.16	67.14%	0.56	2.50	0.06	520
Jan. 90-Dec. 94	5.7551	3.98	0.0884	1.28	102.16%	0.49	2.37	0.07	258
Jan. 95-Dec. 99	4.8533	3.93	0.1449	1.65	87.29%	0.64	2.61	0.05	258

[a] The seemingly unrelated regression model was fitted to estimate the coefficients of $\Delta\delta_t$ and $\ln(S_{t-1}/S_{t-2})$ jointly regressing, respectively, the former variable δ_{t-1} and the latter on its lagged value.
[b] The estimates of η and σ_δ have been annualized.
[c] The Durbin-Watson statistic applies to the first state variable in the S.U.R-model: $\delta_t - \delta_{t-1} = \beta_0 + \beta_1\delta_{t-1} + \varepsilon_t$.
[d] N denotes the number of observations.

The summary of the seemingly unrelated regression in Table 4 verifies that the mean reversion of the convenience yield is strong and fairly stable across the entire sample. In line with previous research indicating significant mean reversion [Gibson et al. (1990) and Bessembinder et al. (1995)]. For our ten-year estimation period any disturbance from the convenience yield mean of 11.83% was corrected within approximately ten weeks. Furthermore, the convenience yield is proven to be highly volatile with large volatility shifts occurring during the sample period. Also, the correlation between spot prices and the convenience yield is high and positive as expected by the theory of storage. So when the supply of oil is limited the spot price is high and the benefits of having an oil inventory are likewise. Figure 3 strongly supports the notion of a stochastic mean reverting convenience yield.

Figure 3
Evolution of the Annualized Oil Convenience Yield

The below figure describes the annualized convenience yield, calculated using Eq. (24) for weekly futures prices between the period Jan.1990-Dec.2000 from the International Petroleum Exchange.

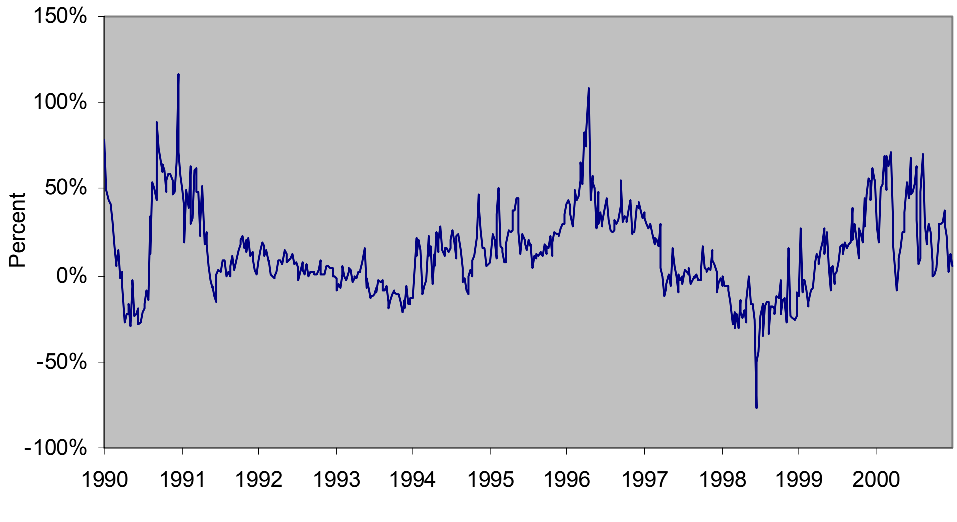


Table 5
Summary Statistics on Pricing Errors in Optimizing the Market Price of Risk, λ

The table indicates the optimized value of the market price of risk. Estimation of the optimal λ was performed by minimizing SSE for contracts with maturities of two to four months over a ten-year period.

Period	λ	*MPE* [a]	*RMSE* [b]	*SSE* [c]	*N*
All contracts [d]	0.1461	-0.01	0.34	185.96	1566
		Period: Jan.1990-Dec.1999			

[a] MPE refers to the mean pricing error in Dollars. Where N denotes number of observations, $\hat{F}$ the theoretical futures price, and F the actual futures price.

$$MPE = \frac{1}{N}\sum_{n=1}^{N}(\hat{F}_n - F_n),$$

[b] RMSE refers to the root mean square error in Dollars.

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\hat{F} - F)^2}$$

[c] SSE refers to the sum of square errors.
[d] All contracts refers to the three nearest maturity contracts following the first futures contract, used as an approximation for the spot price.

The computed λ of 0.1461 was optimized from 1566 weekly futures contracts with two to four months maturity length over our ten-year estimation period. Furthermore, the optimized market price of convenience yield risk led to a within sample MPE of $-0.01 and to an RMSE of $0.34. According to Table 5 the optimized market price of convenience yield risk is positive, implying that investors demand a higher return for bearing convenience yield risk. Strongly disagreeing with classic economic theory. Eq. (5) clearly points out that the presence of convenience yield risk, positively correlated with spot prices, should decrease variability in any portfolio, thereby reducing the investors required return. As a consequence, the risk premium of the convenience should be negative. Gibson et al. (1990) find that the market price of risk for the period of Jan.1984-Nov.1988 for all available contracts is highly negative. In contrast, Schwartz (1997a) detects a positive market price of risk,[31] for the period of Jan.1985-Feb.1995, somewhat higher than our estimate.

5.2 Out of Sample Pricing Performance

Acknowledging the presence of measurement errors in the parameter estimation induces us to test the performance in pricing out of sample futures contracts, between the period Jan.2000-Jul.2000, by again applying Eq. (14). The selected contracts had maturities between two to eight months. Table 6 clearly indicates that the performance of the model for our estimated parameters worsens for contracts with longer maturity. Gibson et al. (1990) make the same observation and prove that the pricing performance is improved using weekly updates of λ. However, in applying real option models we assume a constant λ. Therefore we choose not to update the market price of risk, since we are not interested in improving valuation of short-term contracts. The market price of convenience yield risk is clearly not a constant, although treated as one. Nevertheless, the optimized λ does very well in predicting the term structure, residuals typically range from 2-4%.

Comparing the out of sample pricing errors in Table 6 to the pricing errors of Gibson et al. (1990), indicates that our parameter estimates are clearly better in pricing both the first and

[31] Schwartz is yet to comment this issue.

second period group contracts.[32] The pricing errors using our parameters are typically half the relative size of Gibson et al. (1990). For contracts with maturity of two months our appraisal performs slightly better than Schwartz (1997a) estimates,[33] although the generated estimates are far from the accuracy of Schwartz for longer futures. A plausible explanation could be that we use the parameter appraisal method of Gibson et al. (1990) while the Schwartz study used Kalman Filtering. Furthermore, the former studies use several contracts to estimate λ, among them some contracts of longer maturity. Whereas this study computes the implied market price of risk from contracts with a maximum of four months maturity.

Table 6
Summary of Pricing Errors in Valuing out of Sample Futures Contracts

The table below summarizes the performance of the two-factor model in weekly pricing of futures contracts, for the period of Jan.2000-Jul.2000. The test includes contracts with up to eight months maturity. MPE and RMSE are expressed in U.S. Dollars.

Contracts	*MPE*	*RMSE*	*SSE*	*N*
Futures contract 2	0.10	0.41	4.34	26
Futures contract 3	0.31	0.45	5.19	26
Futures contract 4	0.49	0.60	9.37	26
All first period contracts [a]	0.30	0.49	18.91	78
Futures contract 5	0.67	0.78	16.15	26
Futures contract 6	0.85	0.98	25.17	26
Futures contract 7	1.05	1.18	35.99	26
Futures contract 8	1.25	1.37	49.07	26
All second period contracts [b]	0.95	1.10	126.37	104
	In Percentage of Mean Spot Price [c]			
Futures contract 2	0.38%	1.84%		26
Futures contract 3	1.15%	1.67%		26
Futures contract 4	1.84%	2.24%		26
All first period contracts	1.12%	1.84%		78
Futures contract 5	2.49%	2.94%		26
Futures contract 6	3.19%	3.68%		26
Futures contract 7	3.90%	4.40%		26
Futures contract 8	4.67%	5.13%		26
All second period contracts	3.56%	4.12%		104
	Period: Jan.2000 - Jul.2000			

[a] All contracts in the first period refers to futures 2, 3, and 4.
[b] All contracts in the second period refers to futures 5, 6, 7, and 8.
[c] Mean spot price, $26.77, is calculated as the arithmetical mean of the spot price for the period Jan. 2000 to Jul.2000.

The reason for including MPE in our tests of the pricing ability was to gain insight to whether there is a general over- or underpricing of contracts. Table 6 indicates a general overpricing of all contracts, in line with previous studies.

Nonetheless, the accurate valuation using our parameters indicates that the estimation procedure has been performed properly. Although we are unable to test the parameters in

[32] Taking into account that oil prices were higher during our estimation period and bearing in mind that we include a contract more in our second period group.
[33] Taking into account that oil prices were higher during our estimation period.

pricing long run forward contracts we can presume that the relative performance of our model is unlikely to do much worse, when it comes to valuing long maturity contracts.

Results lend support to Schwartz (1997a) study, indicating a positive market price of convenience yield risk on NYMEX. As a consequence, the notion of a positive market price of risk is strengthened by evidence from two exchanges, over two different estimation periods during the 1990s, and using two different parameter estimation methods. In addition, the pricing ability is clearly superior in studies indicating a positive λ. Clearly this issue demands increased research and the economic logic behind a positive market price of risk should be further explored.

5.3 Value of the Oilfield

Risk-adjusted methods can provide a correct value if all assumptions are reasonable. However, discounting future cash flows with a discount rate always correctly reflecting the future risk of the project is nearby impossible. Petroleum investment in particular involves a great deal of uncertainty and flexibility. As a consequence, the classical DCF model has considerable difficulties in picturing the evolution of risk through the project. Despite these limitations practitioners in the oil industry extensively use it, for reasons of simplicity. By forecasting the oil price through project life the manager arrives at an oilfield value reflecting his believes of future spot prices. Usually managers predict a constant spot price for the entire project. As mentioned previously we used the long-run target price of OPEC, namely \$20 in our valuation. Table 11 in Appendix B indicates an oilfield value of \$51.725 million using the risk-adjusted method. Verifying that the oilfield has a positive NPV and suggesting that investment should be undertaken. The break-even value of the oilfield corresponds to a forecasted spot price of \$15.69 per barrel. So the operator should invest when the oil price rises above the break-even price.

Project value obviously depends on whether executives possess an optimistic view of the future or not, implying that the DCF value is sensitive to the forecasted oil price. Table 7 explicitly declares that project value is very responsive to changes in the long-run oil price.

Table 7
Sensitivity of the DCF Model

The data summarizes the sensitiveness of the risk-adjusted DCF valuation to changes in the spot rate, holding all other parameters constant. All values are reported in million U.S. Dollars.

	DCF Valuation								
Spot price \$	*9*	*12*	*15*	*18*	*20*[a]	*21*	*24*	*27*	*30*
Value	-80.234	-44.048	-7.862	28.325	51.725	64.511	100.697	136.883	172.070
Change	-255.1%	-185.2%	-115.2%	45.2%	0.0%	24.7%	94.7%	164.6%	232.7%

[a] OPEC's long-run target price used as a foundation in our valuation.

Figure 2 shows that far from any long-run oil price can be assumed to be reasonable. Nevertheless, managers have the final word regarding this issue, since they ultimately control all operating decisions.

Any model avoiding subjective projections about future spot prices constitutes a more reliable approach. The certainty equivalent criterion suggests predicting the term structure of forward prices, assuming a predetermined oil extraction rate and certain forecasted costs. Thereby an equivalent position can be taken in Brent forward contracts of matching maturity. Subsequently, resulting cash flows are discounted by a risk-free rate, considering that all cash flows are now known.

The certainty equivalent value implied by Eq. (16) is presented in Fig. 4 for different spot rates and convenience yield levels. We wish to stress that the break-even price of the CEQ valuation is approximately \$19, clearly above the risk-adjusted break-even price. Hence, the decision rule differs for the two methods, which is attributable to the predicted term structure of the Brent price. The term structure is downward sloping for our parameters reported in Table 8, implying that the spot price forecast is more optimistic than the computed term structure. [34]

Table 8
Review of Parameters Used in the Real Option Valuation

This table reports the estimated parameters used in the real option valuation as expressed in Eq. (18). The parameters have previously been reported in Table 2, 3, 4, and 5.

S^a	δ^a	r^a	α	η	$\rho_{S,\delta}$	λ	σ_δ	σ_S
\$24.30	5.20%	5.00%	0.1183	5.2988	0.56	0.1461	67.14%	33.66%

[a] The spot price, convenience yield, and instantaneous risk-free interest rate are all collected on Jan.1, 2001. The current rate of the ten-year government bond approximates the long run risk-free interest rate applied in the estimation of the term structure.

Applying the CEQ approach amounts to an oilfield value of \$41.524 million, slightly below the risk-adjusted value. We derive the real option value of delaying investment by solving the P.D.E. satisfying the oilfield value. Considering the level of the state variables on Jan.1, 2001, implies an option value of \$56.720 million. Indicating only a modest premium over the risk-adjusted value.

Table 9 summarizes the option value for different levels of the spot price and convenience yield. As expected the value of the option to invest increases with the oil price. Furthermore, economic theory proclaims that the value to delay investment should always decrease as the convenience yield increases. Table 9 indicates that increasing the convenience yield for higher Brent prices leads to decreasing oilfield value. However, the value to invest diminishes first at very high levels of the convenience yield, since the impact of lower oil prices is greater than the increased benefit of holding oil.[35]

Table 9 also displays the option premium, referring to the CEQ valuation, for different values of the state variables. For low spot prices the premium is substantial and often exceeds one hundred percent. Confirming the notion that real option value can be substantial even when the CEQ and risk-adjusted value are negative. Increasing Brent prices slowly decrease the premium and it disappears at a spot price of approximately \$27 per barrel, for any convenience yield level. For reasonable ranges of the state variables the premium varies considerably from 20-1000%. Suggesting that overlooking flexibility in petroleum projects can lead to substantial undervaluation of assets and miss-allocation of resources in the economy.

[34] In theory a risk-adjusted value and CEQ value should be equal. Provided that the increase in variability of cash flows is precisely captured by the increase in the discount rate. However, this is usually not achievable in reality.
[35] Although, there is no economic logic behind high benefits of holding crude oil at low oil prices.

Table 9
The Real Option Value of the Oilfield

The table reports oilfield values implied by the real option valuation for different oil price and convenience yield levels. All other parameters are held constant. The option premium is defined as the relative difference between the real option and the certainty equivalent DCF valuation. All values are reported in U.S. Dollars.

Spot price $	*4.00%*	*5.20%*	*11.83%*	*15.00%*	*20.00%*	*30.00%*
9	15 093 000	19 017 000	35 945 000	41 313 000	46 597 000	47 762 000
12	26 207 000	32 767 000	59 016 000	65 964 000	70 646 000	62 782 000
15	32 480 000	38 713 000	67 627 000	75 065 000	76 751 000	56 836 000
18	31 635 000	28 794 000	61 465 000	63 434 000	57 920 000	29 791 000
20	31 542 000	28 267 000	56 296 000	55 261 000	45 178 000	13 607 000
21	36 982 000	44 860 000	65 994 000	64 788 000	52 984 000	15 990 000
24	47 820 000	57 380 000	83 359 000	82 388 000	70 015 000	34 430 000
27	68 204 000	67 612 000	64 368 000	62 831 000	60 426 000	55 684 000
30	97 256 000	96 599 000	92 994 000	91 287 000	88 615 000	83 345 000
Spot price $			*Premium*[36]			
21	266.23%	365.45%	827.53%	994.39%	1208.57%	4334.52%
24	22.14%	48.56%	133.22%	139.67%	117.18%	22.87%
27	0%	0%	0%	0%	0%	0%
30	0%	0%	0%	0%	0%	0%

Figure 4 displays the value created by delaying investment for different convenience yield ranges. Investment is optimal at a Brent price of $26.72 per barrel for both the actual convenience yield of 0.0520 and the mean convenience yield. Comparing the values for different convenience yields indicates that investment should be undertaken for oil price ranges between 25-27 U.S. Dollars. Strongly contrasting with the decision rule suggested by the CEQ and the risk-adjusted DCF valuation. Indicating that investment should be undertaken when Brent prices are 18-19 U.S. Dollars and 15.69 U.S. Dollars respectively.

The option curves are not exponentially upward sloping, an explanation for this inconsistency could be that the oilfield generates different quantities of oil each year. Thereby cash flows vary considerably to the extent that a smooth solution to the P.D.E. is not achievable. Previous studies have used yearly fixed annuities to model cash flows, enabling a smooth solution of oilfield value. We are of the opinion that these simplifications unable a realistic description of the inherent flexibility in the petroleum industry, where a vast majority of extractable oil is often developed in the earliest years of the project. Consequently, our option curve, although not smooth, describes the flexibility in a more realistic way.

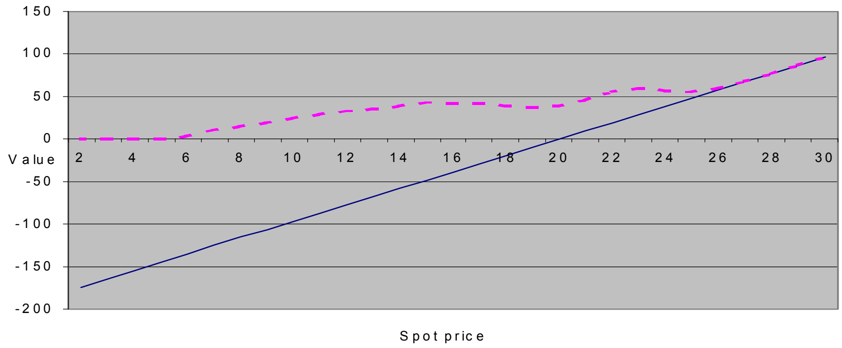
We conclude by saying that the premium today over the CEQ estimate is approximately 37%, whereas the premium over the risk-adjusted value is 10%. Still, for lower oil prices absolute premiums will be larger since both cash flow methods then indicate a negative value. Drawing any conclusions for higher oil prices is however difficult, since managers predictions about future Brent prices will be decisive to whether there exists an option premium. The only valid conclusion is that the premium will slowly disappear as the spot price rises. Nonetheless, whether there is a premium or not does not preclude the usage of real option models, since they maximize value by accounting for the strategic aspect of investing.

[36] Given that the CEQ value can be negative the relative premium becomes misleading, therefore we only present the premium whenever CEQ is positive.

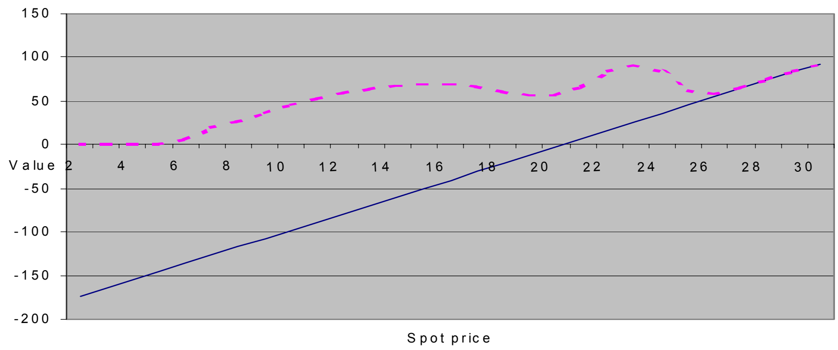
Figure 4
Real Option Valuation vs. Certainty Equivalent Value

These three figures compare the evolution of the real option value and the certainty equivalent value, using different spot prices and a constant convenience yield. Option values are pictured by the dotted line, whereas the unbroken line presents the CEQ value. Any value difference should be interpreted as the option premium in absolute terms. Oilfield values are reported in million U.S. Dollars.

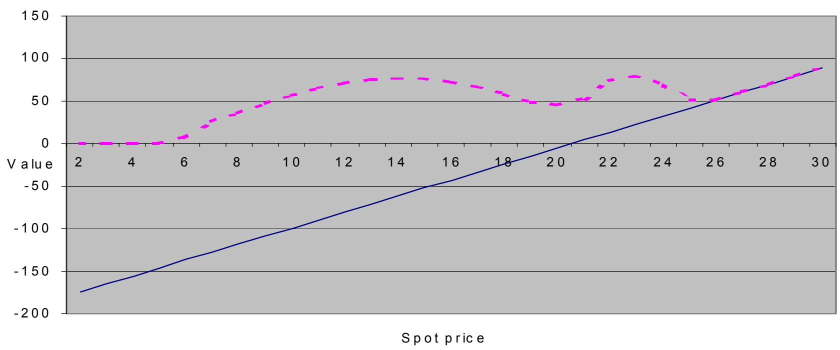
Convenience yield of 0.0520



Convenience yield of 0.1183



Convenience yield of 0.2000



5.4 Sensitivity Analysis

To test the applicability of all three models a sensitivity analysis is carried out. Table 10 indicates that the CEQ valuation is more responsive to changes in all parameters accept the volatility of the state variables.

Table 10
Sensitivity of the Real Option and DCF Model

This table reports the sensitivity of the models to changes in real option (R.O.) parameters. The value on Jan.1, 2001 is calculated using the corresponding spot price of $24.3 and convenience yield of 0.052, holding all other parameters constant. The DCF valuation is calculated according to our earlier assumption using a constant spot price of $20. Thereby it is only responsive to one parameter. All values are reported in million U.S. Dollars.

	R.O. Valuation		*CEQ Valuation*		*Premium*	*DCF Valuation*	
r	*Value*	*Change*	*Value*	*Change*		*Value*	*Change*
4.00%	61.462	8.36%	37.789	-8.99%	62.65%	58 474	13.05%
5.00%[a]	56.720	0.00%	41.524	0.00%	36.60%	51.725	0.00%
6.00%	55.266	-2.56%	44.991	8.35%	22.84%	46 756	-9.61%
σ_δ							
37.14%	114.620	102.08%	38.485	-7.32%	197.83%	51.725	0.00%
67.14%[a]	56.720	0.00%	41.524	0.00%	36.60%	51.725	0.00%
97.14%	47.842	-15.65%	47.842	15.22%	0.00%	51.725	0.00%
σ_S							
20.66%	51.026	-10.04%	51.026	22.88%	0.00%	51.725	0.00%
33.66%[a]	56.720	0.00%	41.524	0.00%	36.60%	51.725	0.00%
46.66%	89.079	57.05%	32.474	-21,79%	174.31%	51.725	0.00%
$\rho_{\delta S}$							
0.36	54.473	-3.96%	50.294	21.12%	8.31%	51.725	0.00%
0.56[a]	56.720	0.00%	41.524	0.00%	36.60%	51.725	0.00%
0.76	69.623	22.75%	33.140	-20.19%	110.09%	51.725	0.00%
α							
7.83%	86.375	52.28%	86.375	108.01%	0.00%	51.725	0.00%
11.83%[a]	56.720	0.00%	41.524	0.00%	36.60%	51.725	0.00%
15.83%	42.672	-24.77%	5.225	-87.42%	716.69%	51.725	0.00%
η[37]							
1.2988	163.970	189.80%	163.970	294.88%	0.00%	51.725	0.00%
5.2988[a]	56.720	0.00%	41.524	0.00%	36.60%	51.725	0.00%
9.2988	45.337	-20.07%	37.313	-10.14%	21.50%	51.725	0.00%
λ							
-1.000	41.624	-26.61%	-62.194	-249.78%	-	51.725	0.00%
-0.500	46.781	-17.52%	-25.698	-161.89%	-	51.725	0.00%
0.1461[a]	56.720	0.00%	41.524	0.00%	36.60%	51.725	0.00%
0.500	92.501	62.08%	92.501	122.77%	0.00%	51.725	0.00%
1.000	190.500	235.86%	190.500	358.77%	0.00%	51.725	0.00%

[a] Estimated value of the parameter.

[37] The reason for presenting such a wide range of the mean reversion factor is the variety of different estimates in former studies.

Changing the risk-free interest rate has a significant affect on the risk-adjusted value, through the indirect effect on the WACC. The option value however, is less sensitive to changes in the long-run bond yield. Oilfield value decreases modestly with rising interest rates, contradicting the logic of the Black and Scholes formula. As pointed out by Fernandez (2001) a frequently made error in real option valuation is to assume that value increases with interest rates. In this case the cash flow structure of the project is affected more than the risk-free growth rate of the oil price.

The standard deviation of the convenience yield has a notable affect only when benefits are decreasing. Decreases in variability of the benefits will all else equal, increase the option value of the oilfield. Uncertainty in future spot prices has a contrary effect, increasing the option value and the premium for rising volatility. Table 10 displays that lower spot rate volatility will hardly reduce value at all, although, the premium will diminish. We wish to stress that the positive correlation of the two state variables reduces variability in the oilfield value. Thereby the value becomes less sensitive to increased uncertainty.

Previous studies assessments of oil price volatility correspond closely to the estimate of this study, in addition volatility shifts for subperiods are small, as shown by Table 3. Consequently, we argue that the often criticized assumption of constant spot rate volatility is in fact one of the most reasonable simplifications in the model.

The mean convenience yield, α, has a significant effect on value since it has a strong influence on the term structure of futures prices. Accordingly, a decrease in α implies higher prices of long-term contracts, which increases value. Moreover, lowering the mean reversion factor results in large value appreciation of the oil venture. As a result, the option premium diminishes for lower rates of mean reversion. Variability in the market price of risk, λ, is also crucial for project value. Decreasing the market price of risk from low levels has a modest effect, while drastic increases have a tremendous effect on value.

Summarizing the applicability of the real option approach indicates that accurate appraisal of mean reversion parameters is crucial to arrive at reliable values.

5.5 Reliability and Validity

The input data of our study are reliable in view of the fact that they are provided by Reuters and IPE. In addition, all illiquid contracts on the IPE have been abolished in our study, suggesting that the historic market prices are trustworthy.

Real option models typically demand estimation of several parameters, some of which are highly volatile. Thereby it becomes necessary to test the model in pricing futures and forward contracts, to enable conclusions concerning the reliability of the results. Tests of the pricing accuracy are satisfying and lead us to conclude that the estimation procedure has been performed properly. Nonetheless, the strength of the results in valuing long-term contingent claims rests on the assumption that parameter values of short-term futures and long-term forwards are similar. Clearly a questionable assumption, emphasizing the need to further explore implications of estimating parameters from contracts with shorter length than the project. Furthermore, the real option procedure implies making assumptions that sometimes contradict reality. In particular, parameters are estimated from historical data not necessarily describing the future. Also, some of the parameters are assumed constant over the life of the project. Namely the risk-free interest rate, the volatility of the state variables, correlation, and the market price of risk. The latter assumption is clearly arguable. As a final point security trading is assumed to be continuous and involve no transaction costs.

6. SUMMARY AND CONCLUSIONS

We examine whether the value of a North Sea oilfield differs depending on if real option valuation or DCF valuation is used. More specifically, we account for the option to delay investment, assuming that model uncertainty is inherent from the spot price of Brent and the convenience yield. Findings indicate an option value of $56.720 million, implying a premium over the certainty equivalent value of approximately 37%. Although performing an additional risk-adjusted valuation indicates a more modest premium of 10%. Emphasizing the need to complement the real option method with a DCF valuation expressing the managers believes of future oil prices.

In general the real option premium varies considerably and ranges from 20-1000%, for reasonable values of the spot price and the convenience yield. However, it is impossible to draw any general conclusions about the premium over a risk-adjusted method, since this method is entirely dependent on the manager's outlook for future spot prices. Nevertheless, a substantial premium definitely exists for pessimistic price predictions, implying that information costs incurred applying option methods are negligible in relation to the discovered hidden values.

Acknowledging that the DCF method presents similar values to the option approach for high spot prices does not mean that the option criterion can be neglected. Bearing in mind that it also has implications for the strategic decision of when to optimally invest. The risk-adjusted approach suggests that investment is optimal whenever oil prices surpass $15.69 whereas the real option analysis suggests production at prices above $26.72 per barrel. Reasonable values of the spot rate and the convenience yield imply an optimal investment price range of 25-27 Dollars per barrel. Accounting for the option to delay has considerable implications for both valuation and strategy. The two cash flow methods suggest that investment should be undertaken on Jan.1, 2001 while the real option approach indicates that development should be delayed further.

The main reason for not using the real option method is the complexity and the trouble of describing results to clients. Limitations of the real option framework are also inherent in the parameter estimation and the simplified view of reality. For example, petroleum properties are generally leased by several operators who have to agree on various decisions. In addition, the decision to invest can be influenced by the state of the firms other assets, in case the company is experiencing a shortage of funds. Moreover, geological conditions can also play an important part in investment decisions.

We also find evidence of a positive market price of convenience yield risk on the IPE, strongly disagreeing with economic theory. Findings thereby lend support to Schwartz (1997a) study, indicating a positive market price of convenience yield risk on NYMEX. As a consequence, the notion of a positive market price of risk is strengthened by evidence from two exchanges, using two different parameter appraisal methods over two different estimation periods, during the 1990s.

Since we did not consider all options available to an oilfield leaseholder an interesting extension to this study would be to include the abandonment option to further analyze the option premium. Another important area of research is the implications for valuation and hedging by modeling oil price changes as a mean reverting jump process. Furthermore, the consequence of using short-term contracts in the valuation of long-term assets needs more attention. Finally, we believe that more research is required focusing on whether the market capitalization of petroleum companies can be explained by standard DCF methods.

Appendix A

Generalized Ito's Lemma: $df = \left(\sum_i \frac{\partial f}{\partial x_i} a_i - \frac{\partial f}{\partial \tau} + \frac{1}{2} \sum_i \sum_j \frac{\partial^2 f}{\partial x_i \partial x_j} b_i b_j \rho_{ij} \right) dt + \sum_i \frac{\partial f}{\partial x_i} b_i dz_i$

where,

f = function depending on variables x_1, x_2, , ... x_n and time t.
x_i = state variable following an Ito process with drift a_i and standard deviation b_i

$$dS = (\mu - \delta) S dt + \sigma_1 S dz_1 \quad \textbf{(1)}$$
$$d\delta = [\eta(\alpha - \delta)] dt + \sigma_\delta dz_\delta \quad \textbf{(2)}$$

Define f as any financial security dependent on the spot price S and convenience yield δ, the change in the security is then described by applying Ito's Lemma

$$df = \left[\frac{\partial f}{\partial S}(\mu - \delta)S + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta)) - \frac{\partial f}{\partial \tau} + \frac{1}{2}\left(\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 + 2\frac{\partial^2 f}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\, S \right) \right] dt +$$
$$+ \frac{\partial f}{\partial S}\sigma_S S dz_S + \frac{\partial f}{\partial \delta}\sigma_\delta dz_\delta \quad \textbf{(3)}$$

Subsequently we express the relative change by dividing both sides by f

$$\frac{df}{f} = \left[\left(\frac{\partial f}{\partial S}(\mu - \delta)S + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta)) - \frac{\partial f}{\partial \tau} + \frac{1}{2}\left(\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 + 2\frac{\partial^2 f}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\, S \right) \right) / f \right] dt +$$

$$+ \left(\frac{\partial f}{\partial S}\sigma_S S / f \right) dz_S + \left(\frac{\partial f}{\partial \delta}\sigma_\delta / f \right) dz_\delta \quad \textbf{(4)}$$

For simplicity define

$$\kappa = \left(\frac{\partial f}{\partial S}(\mu - \delta)S + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta)) - \frac{\partial f}{\partial \tau} + \frac{1}{2}\left(\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 + 2\frac{\partial^2 f}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\, S \right) \right) / f \quad \textbf{(5)}$$

$$W_1 = \frac{\partial f}{\partial S}\sigma_S S / f \quad \textbf{(6)}$$
$$W_2 = \frac{\partial f}{\partial \delta}\sigma_\delta / f \quad \textbf{(7)}$$

Brennan and Schwartz (1979) propose forming a portfolio P by investing amounts of x_1, x_2, and x_3 in three bonds of maturity τ_1, τ_2, and τ_3. The rate of return on this portfolio is

$$\frac{df}{f} = [x_1\kappa + x_2\kappa + x_3\kappa]dt + [x_1 W_1 + x_2 W_1 + x_3 W_1]dz_1 + [x_1 W_2 + x_2 W_2 + x_3 W_2]dz_2 \quad \textbf{(8)}$$

To achieve a non-stochastic rate of portfolio return we need to adjust portfolio proportions so that the coefficients of dz_1 and dz_2 in Eq. (8) are zero

$$x_1 W_1 + x_2 W_1 + x_3 W_1 = 0 \tag{9}$$

$$x_1 W_2 + x_2 W_2 + x_3 W_2 = 0 \tag{10}$$

To prevent arbitrage the return on this portfolio must be risk-less over short time intervals, that is the return is equal to *r*, the instantaneous risk-free interest rate. As a consequence, the portfolio risk premium is zero

$$x_1(\kappa(\tau_1) - r) + x_2(\kappa(\tau_2) - r) + x_3(\kappa(\tau_3) - r) = 0 \tag{11}$$

The no arbitrage condition Eq. (11) and the two zero risk conditions Eq. (9) and (10) will have a solution only if

$$\kappa - r = \lambda_S W_1 + \lambda\, W_2 \tag{12}$$

λ = market price of convenience yield risk

Subsequently, Eq. (5), (6), and (7) are substituted into Eq. (12) resulting in

$$\left[\left(\frac{\partial f}{\partial S}(\mu - \delta)S + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta)) - \frac{\partial f}{\partial \tau} + \frac{1}{2}\left(\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 + 2\frac{\partial^2 f}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\; S\right)\right) / f\right] - r =$$

$$= \lambda_S \frac{\partial f}{\partial S}\sigma_S S / f + \lambda \frac{\partial f}{\partial \delta}\sigma_\delta / f \tag{13}$$

Simplifying and moving all terms to the left hand side

$$\frac{\partial f}{\partial S}S((\mu - \delta) - \lambda_S \sigma_S) + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta) - \lambda \sigma_\delta) + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{1}{2}\frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 +$$

$$+ \frac{\partial^2 f}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\; S - \frac{\partial f}{\partial \tau} - rf = 0 \tag{14}$$

Keeping in mind that

$$\mu - \delta - \lambda_S \sigma_S = r - \delta \tag{15}$$

Consequently, Eq. (14) thereby results in the below P.D.E. satisfying any derivative dependent on the evolution of the Brent spot rate and the convenience yield

$$\frac{\partial f}{\partial S}S(r - \delta) + \frac{\partial f}{\partial \delta}(\eta(\alpha - \delta) - \lambda \sigma_\delta) + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma_S^2 S^2 + \frac{1}{2}\frac{\partial^2 f}{\partial \delta^2}\sigma_\delta^2 + \frac{\partial^2 f}{\partial S \partial \delta}S\sigma_S \sigma_\delta \rho\; - \frac{\partial f}{\partial \tau} - rf = 0$$

Appendix B

Table 11
Discounted Cash Flow Valuation 2001-2013 ($)

The table reports the estimated future cash flows using a constant oil price of $20 and a exchange rate estimated through interest rate parity, meaning that forward exchange rates depend on the interest rate difference between the countries. All cash flows are received in the beginning of each year.

Year	*Barrels per Year*	*Oil Price*	*Forw. Rate ($/£)*	*Operating exp.[c] ($)*	*FCF*	*Tax*	*After Tax DCF ($)*
2001				-132 662 000[a]	-132 662 000		-132 662 000
2002							
2003							
2004	7 793 487	20	1.483	-25 542 000	130 328 000	-19 550 000	91 155 000
2005	3 312 984	20	1.480	-12 834 000	53 426 000	-8 010 000	35 016 000
2006	1 988 994	20	1.477	-9 129 000	30 650 000	-4 600 000	18 825 000
2007	1 489 489	20	1.473	-7 761 000	22 029 000	-3 300 000	12 678 000
2008	1 191 591	20	1.470	-6 992 000	16 839 000	-2 530 000	9 082 000
2009	1 074 237	20	1.467	-6 723 000	14 762 000	-2 210 000	7 460 000
2010	965 911	20	1.463	-6 491 000	12 827 000	-1 920 000	6 075 000
2011	869 621	20	1.460	-6 308 000	11 084 000	-1 660 000	4 919 000
2012	625 886	20	1.457	-6 138 000	6 380 000	-96 000	2 653 000
2013	156 472	20	1.454	-9 133 000[b]	-6 003 000	0	-2 752 000
Net Present Value							$ 51 725 000

[a] Capital expenditure in Dollars made at the beginning of year 2001.
[b] Abandonment cost year 2013.
[c] Operating expenses in Dollars were computed using the above forward rates.

Table 12
The Beta Estimation[a]

This table reports the Betas collected from Bloomberg and Yahoo finance. The Beta used in the DCF valuation is an arithmetic mean of the collected Betas.

Company	*Bloomberg*	*Yahoo*
Anglo Siberian	0.50	0.48
BG Group Plc	0.53	0.36
BP Amoco	0.83	0.97
Cairn Energy	0.57	0.40
Edinburgh Oil and Gas	0.58	0.95
Enterprise Oil	0.68	0.46
JKX Oil and Gas	0.45	0.62
Northern Petroleum	0.90	1.19
Premier Oil Plc	0.62	0.60
Shell	0.93	1.28
Soco Intl.	0.56	0.53
Mean beta		0.70

[a] Betas were collected on Mar.10, 2002.

Appendix C

The syntax in Matlab has been separated into nine parts to create a simple overview.
Main: Uses the previously defined partial derivatives and performs the numerical solution.
Rhside: Summarizes the boundary conditions and defines the size of the steps.

CEQ_Fun.m: $-k+\sum_{T}^{N} e^{-rt}\left[(Q_t F(S,\delta,T)-C_t X_t)(1-h_t)+h_t D_t\right]$

V_d_Fun.m: $\frac{\partial V}{\partial \delta}(\eta(\alpha-\delta)-\lambda\sigma_\delta)$

V_dd_Fun.m: $\frac{1}{2}\frac{\partial^2 V}{\partial \delta^2}\sigma_\delta^2$

V_S_Fun.m: $\frac{\partial V}{\partial S}(r-\delta)S$

V_SS_Fun.m: $\frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma_S^2 S^2$

V_Sd_Fun.m: $\frac{\partial^2 V}{\partial S \partial \delta}\sigma_S\sigma_\delta\rho\ S$

V_Fun: rV

Main:

```
E=[];
parameters=                 % parameters
  [0                        % 1) : D_t
  0.05                      % 2) : r
  0.1183                    % 3) : alpha
  5.2988                    % 4) : eta
  0.56                      % 5) : rho
  0.1461                    % 6) : lambda
  0.6714                    % 7) : sigma_delta
  0.3366                    % 8) : sigma_S
  132662000                 % 9) : k
  0                         % 10) : C_t1
  0                         % 11) : C_t2
  0                         % 12) : C_t3
  25542000                  % 13) : C_t4
  12834000                  % 14) : C_t5
  9129000                   % 15) : C_t6
  7761000                   % 16) : C_t7
  6992000                   % 17) : C_t8
  6723000                   % 18) : C_t9
  6491000                   % 19) : C_t10
  6308000                   % 20) : C_t11
  6138000                   % 21) : C_t12
  9133000                   % 22) : C_t13
  0                         % 23) : Q_t1
  0                         % 24) : Q_t2
  0                         % 25) : Q_t3
  7793487                   % 26) : Q_t4
```

```
  3312984                       % 27) : Q_t5
  1988994                       % 28) : Q_t6
  1489489                       % 29) : Q_t7
  1191591                       % 30) : Q_t8
  1074237                       % 31) : Q_t9
  965911                        % 32) : Q_t10
  869621                        % 33) : Q_t11
  625886                        % 34) : Q_t12
  156472                        % 35) : Q_t13
  0.15                          % 36) : h_t
  12                            % 37) : N
  1];                           % 38) : X_t
for refin=5
for refint=1
% grid;
delta_min=0;
delta_max=1;
S_max=45;
S_min=0;
N_delta=2^refin+1;
N_S=2^refin+1;
hdelta=(delta_max-delta_min)/(N_delta-1);
hS=(S_max-S_min)/(N_S-1);
dtau=0.001/refint;
[delta,S]=meshgrid(delta_min+hdelta:hdelta:delta_max-hdelta,S_min+hS:hS:S_max-hS);
delta((N_S-1)/4,(N_delta-1)/4);
f=zeros(size(S));
tau=0;
CEQ=CEQ_Fun(delta,S,parameters);
CEQ=CEQ.*(CEQ>0);
for tau=0:dtau:11000*dtau/refint-dtau
  s1=dtau*feval('rhside',f,tau,hdelta,hS,delta,S,parameters);
  s2=dtau*feval('rhside',f+s1/2,tau+dtau/2,hdelta,hS,delta,S,parameters);
  s3=dtau*feval('rhside',f+s2/2,tau+dtau/2,hdelta,hS,delta,S,parameters);
  s4=dtau*feval('rhside',f+s3,tau+dtau,hdelta,hS,delta,S,parameters);
  f=f+(s1+2*s2+2*s3+s4)/6;
  f=f.*(f>0);
  f=f.*(CEQ<f)+CEQ.*(CEQ>f);
  subplot(1,2,1)
  surf(delta,S,f);
  title(tau+dtau)
  drawnow
  subplot(1,2,2)
  surf(delta,S,CEQ_Fun(delta,S,parameters));
  title(tau+dtau)
  drawnow
end
end
end
```

Rhside:

```
function dfdtau=rhside(f,t,hdelta,hS,delta,S,parameters);
k=parameters(9);
% expanding the solution matrix with the BC.
bc1=[0 S(1,:) 0]*0;   % S=S_min
bc2=S(:,1)*0;         % delta=delta_min
bc3=CEQ_Fun(delta(1,end)+hdelta,S(:,1),parameters)+k;     % delta=delta_max
bc4=CEQ_Fun([delta(1,1)-hdelta delta(1,:) delta(1,end)+hdelta],S(end)+hS,parameters)+k; %
S=S_max
f=[bc1;[bc2 f bc3];bc4];

%% f=V

V=f(2:end-1,2:end-1);
V_dd=(f(2:end-1,3:end)-2*V+f(2:end-1,1:end-2))/(hdelta^2);
V_SS=(f(3:end,2:end-1)-2*V+f(1:end-2,2:end-1))/(hS^2);
V_Sd=(f(3:end,3:end)-f(1:end-2,3:end)+f(1:end-2,1:end-2)-f(3:end,1:end-2))/(4*hS*hdelta);
V_d=(f(2:end-1,3:end)-f(2:end-1,1:end-2))/(2*hdelta);
V_S=(f(3:end,2:end-1)-f(1:end-2,2:end-1))/(2*hS);

dfdtau=V_SS*V_SS_Fun(t,delta,S,parameters)+...
      V_dd*V_dd_Fun(t,delta,S,parameters)+...
      V_Sd*V_Sd_Fun(t,delta,S,parameters)+...
      V_S*V_S_Fun(t,delta,S,parameters)+...
      V_d*V_d_Fun(t,delta,S,parameters)+....
      V*V_Fun(t,delta,S,parameters)+...
      F_Fun(t,delta,S,parameters);
```

CEQ_Fun.m:

```
function F=CEQ_Fun(delta,S,parameters);
k=parameters(9);
N=parameters(37);
r=parameters(2);
X_t=parameters(38);
h_t=parameters(36);
D_t=parameters(1);
alpha=parameters(3);
lambda=parameters(6);
sigma_S=parameters(8);
eta=parameters(4);
sigma_delta=parameters(7);
rho=parameters(5);
t=(0:N);
alpha_hat=alpha-lambda*sigma_delta/eta;
CEQ=0*S-k;
for n=1:length(t)
     T=t(n);
  A=(r-alpha_hat+sigma_delta.^2/eta^2/2-sigma_S.*sigma_delta*rho/eta)*T+...
  sigma_delta.^2/4*(1-exp(-2*eta*T))/eta^3+...
  (alpha_hat*eta+sigma_S.*sigma_delta*rho-sigma_delta.^2/eta)*(1-exp(-eta*T))/eta^2;
```

```
   C_t=parameters(9+n);
  Q_t=parameters(22+n);
CEQ=CEQ+exp(-r*T)*((Q_t*S.*exp(-delta*(1-exp(-eta*T))/eta+A)-C_t*X_t)*(1-
h_t)+h_t*D_t);
end
F=CEQ;
```

V_d_Fun.m:

```
function F=V_d_Fun(t,delta,S,parameters);
eta=parameters(4);
alpha=parameters(3);
lambda=parameters(6);
sigma_delta=parameters(7);
F=(eta*(alpha-delta)-lambda*sigma_delta);
```

V_dd_Fun.m:

```
function F=V_dd_Fun(t,delta,S,parameters)
sigma_delta=parameters(7);
F=sigma_delta^2/2;
```

V_S_Fun.m:

```
function F=V_S_Fun(t,delta,S,parameters);
r=parameters(2);
F=(r-delta).*S;
```

V_SS_Fun.m:

```
function F=V_SS_Fun(t,delta,S,parameters)
sigma_S=parameters(8);
F=sigma_S^2*S.^2/2;
```

V_Sd_Fun.m:

```
function F=V_Sd_Fun(t,delta,S,parameters);
sigma_S=parameters(8);
sigma_delta=parameters(7);
rho=parameters(5);
F=sigma_S*sigma_delta*rho*S;
```

V_Fun:

```
function F=V_Fun(t,delta,S,parameters);
r=parameters(2);
F=-r;
```

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