

A Note on Valuation of Companies with Growth Opportunities

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ABSTRACT

Each company faces day to day investment opportunities. Just by staying in business the company is taking a decision of reinvesting. The question arising for those managers who have the responsibility of allocating capital is the criteria they should use to differentiate between investment alternatives. The most proven, traditional and popular method of valuation is Discounted Cash Flow (henceforth DCF), which provides comparable information. This method requires both the assesment of expected future cash flows and a risk adjusted rate (used in the discount coefficient).

Besides the current business the company is in, it can also face horizontal or vertical growth opportunities should events unfold favourable. Given the existence of these options for contingent or future growth, what would therefore be the value of the company?

I Introduction

I.1 Flexibility on decisions

Allocating resources in a company does not imply a rigid plan of activities, but a set of decisions conditional upon new information arriving, so decisions are sequential and cannot be fully planned in advance. This means decisions are taken as uncertainty unfolds, at the right moment. The manager has some times the flexibility to delay taking some decisions until he obtains more information. As long as this flexibility does not cause a loss to the company, it has a positive value. These decisions the manager faces when allocating resources can be grouped into the following broad categories

- Growth decisions
- Contraction or even abandonment decisions
- Delay decisions

In all cases the company faces options that can be exercised only if events turn out to be favourable¹. This reflects the right (not the obligation) the management team has. This flexibility (or the options it implies) has value, is non trivial for the value of the company. For example, two companies identic in everything but with a particular customer portfolio each, which allows one company to corss sell more products or services should market conditiones turn favourable, cannot be worth the same.

On this paper I shall focus the analysis on growth options, its structure and valuation.

¹ Otherwise the company can let the option expire and not exercise it.

I.2 Assumptions

The following assumptions will be made to make the world more tractable:

- The typical investor is risk averse,
- Capital markets are complete
- The information set is the same for all investors (information is symmetric).
- Growth options embedded in projects take the form of the European derivatives, where early exercise is not allowed,
- The risk free rate is non stochastic and given,
- The value of the business in each state of the nature is known,
- There is an appropriate way of obtaining the risk adjusted rate of return reflecting properly risk preferences of the investors,
- The probabilities of each state of the nature are known.
- Finally, in a binomial world when moving the value of probabilities, volatility changes. We shall ignore this effect on the risk adjusted rate of return.

II Valuation Techniques

II.1 The traditional methodology

This method accounts for the calculation of the expected value of future cash flows, discounting them using a risk adjusted discount rate², intended to show the preferences towards risk of the average investor. In terms of a discrete distribution of probabilities, the present value of a one period project can be shown to be

$$V_t = \sum \frac{\Sigma p_{i,t+1} * V_{i,t+1}}{(1+k)^{t+1}}$$

where $V_{i,t+1}$ represents the possible values the project can undertake in each state of the nature i ,

p_i accounts for the likelihood of each state of the nature

k is the equilibrium risk adjusted rate of return.

² To the purpose of obtaining the appropriate rate, a standard Capital Asset Pricing Model (CAPM) of the form

$$E(R_i) = r_f + b_i * (E(R_m) - r_f)$$

can be used, where the left hand side represents the expected return the project has to earn, and the right hand side accounts for two terms, r_f for the risk free rate, and a risk premia. The coefficient b reflects the degree of covariance between the return of an asset in a similar risk class and the market (well diversified portfolio) rate of return. According to the model, in equilibrium the investor pays only for the risk he cannot diversify by himself.

II.2 Contingent claim analysis

Alternatively, in a complete capital market an investor can pay a price π_i at time t to obtain a pure asset which pays at $t+1$ a dollar should state i of the nature happens and zero otherwise. Investors wanting to ensure one dollar in every state of the nature will have to buy a complete set of pure assets paying for it the sum of the prices of each pure asset ($\sum \pi_i$). The portfolio thus obtained will have the property of being riskless (the payoff of such a portfolio is the same regardless of the state of the nature), hence in equilibrium and to rule out arbitrage opportunities, the return of such a portfolio has to be equal to the risk free return. Lets call r to the risk free rate, thus

$$\sum \pi_i = 1/(1+r)$$

Therefore, in equilibrium an asset that pays V_i dollars in the state of the nature i and zero otherwise has to be worth $\pi_i V_i$. It follows that at $t+1$ an asset with payoffs of V_i in each state of the nature i is worth

$$V_t = \sum \pi_i V_{i,t+1} \text{ at } t$$

Working on this formula, multiplying and dividing by $\sum \pi_i$ and redistributing, we obtain

$$V_t = \sum \pi_i V_{i,t+1} \frac{\sum \pi_i}{\sum \pi_i} = \sum \frac{\pi_i}{\sum \pi_i} * V_{i,t+1} * \sum \pi_i$$

and making

$$\tilde{p}_i = \frac{\pi_i}{\sum \pi_i}$$

and

$$\sum \pi_i = \frac{1}{(1+r)}$$

we have that the value of a such a project V is shown to be:

$$V_t = \sum \tilde{p}_i * V_{i,t+1} * \frac{1}{(1+r)}$$

In other words, the value is the expected value of the payoffs using a synthetic probability distribution, discounted at the risk free rate. It can be easily seen that this new probability distribution satisfies all the requirements of any probability distribution (non negative values, the sum of all at a certain time adding up to one, etc). We have valued the project using the risk free rate in the discount factor, just as if the investor was risk neutral. Nevertheless, the value of the project V_t obtained is the same under the two alternatives.

III Growth Option

III.1 Growth decisions

Growth decisions that a manager can face are:

- expand business vertically (buy out or set up business within the value chain)

- expand business horizontally (buy out or set up business not directly related with the value chain)
- expand the business (gain market share)

A company can face a project which allows, in case events turn out to be good and circumstances are appropriate, to expand further. Even though this decision is not taken at the outset, the current value of the firm should reflect this option³.

Continuing with the valuation structure described above, we assume that in a particular state of the nature j at $t+1$, the investor has the opportunity to undertake further investments with expected cash flows of n times the value of the project at this moment ($nV_{i,t+1}$) by paying a cost K . This means the investor will pay the cost K only if $nV_{j,t+1} \geq K$, or $nV_{j,t+1} - K \geq 0$ ⁴. If this inequality does not hold, the investor would be paying more than what the asset is worth. It can be seen that the investor would buy the asset (exercise her option to expand) only in those states of the nature where V_{t+1} is sufficiently high. In formula, the payoff or value of business in each state of the nature becomes

$$V_{i,t+1} + \text{Max}(nV_{i,t+1} - K, 0)$$

And the current value of business is thus (we shall label the current value of this asset $V_{t,A}$)

$$V_{t,A} = \frac{\sum p_i * (V_{i,t+1} + \text{Max}(nV_{i,t+1} - K, 0))}{1 + k}$$

The value (as shown before), can also be obtained using the contingent claim analysis or risk neutral valuation. Now we shall label the value obtained by this method $V_{t,B}$

³ See Section VII for a detailed analysis

⁴ We avoid the analysis of agency problems between managers and shareholders.

$$V_{t,B} = \frac{\sum \tilde{p}_i * (V_{i,t+1} + \text{Max}(nV_{i,t+1} - K, 0))}{1 + r}$$

where synthetic (or risk neutral) probabilities derived previously are used.

It is the objective of this paper to demonstrate that for bussines with growth options embedded (representing flexibility for further expansion), the DCF method overvalues the true value of the option. Should this hypothesis be verified, would mean that for some cases traditional valuation methodology has to be adjusted to reflect the overestimation.

III.2 Two states of the nature, one period model

Consider the simplest case, where we have two states of the nature at t+1, and the project value V can adopt two possible values, one for each state i. Assume there exists a risk free asset which pays a return of r. The likelihood of state 1 is given by p, while likelihood of state 2 is the complement 1-p. According to the traditional method of valuation, an asset of such features would be worth

$$V_t = (p_1 * V_{1,t+1} + p_2 * V_{2,t+1}) * \frac{1}{1 + k}$$

where k is a representative risk adjusted rate of return. Consistently with what has been seen above, we can find a synthetic probability \tilde{p} based on the values V_1 y V_2 , through which we obtain an expected value of V at t+1. Discounting this expected value by using the risk free rate, the same value V_t derived by traditional methodology obtains.

This probability distribution based on \tilde{p} comes out from setting the return of the asset equal to the risk free return, and changing the density mass of the probability distribution at each point of the possible values V at $t+1$. The probabilities found this way are consistent with the current or spot value of the asset. Resuming, the changes introduced are;

- take the current value of the asset
- set its return equal to the risk free return
- find the probabilities associated to this new expected value by changing the probability mass at each point of the possible values of V .

In formula

$$V = [\tilde{p} * V_1 + (1 - \tilde{p}) * V_2] \frac{1}{(1 + r)}$$

rearranging terms

$$(1 + r) * V = \tilde{p} * V_1 + (1 - \tilde{p}) * V_2$$

is an equation with one unknown variable, which can be easily solved for

$$\tilde{p} = \frac{V * (1 + r) - V_2}{V_1 - V_2}$$

and

$$(1 - \tilde{p}) = \frac{V_1 - (1 + r) * V}{V_1 - V_2}$$

Armed with this synthetic probability, V_t obtains by taking the expected value and discounting it to the risk free rate of return. As it was shown, the value V_t remains the same under the two methodologies, but in the second case the value is obtained as if the investor was neutral to risk.

We capture the random structure of V from the parameters V_1, V_2 and $(1+r)$, which in turn are used to obtain the set of synthetic probabilities \tilde{p} consistent with V_t .

Now suppose the project has growth options embedded. As it was exemplified before, the investor has the right to pay a cost of K to reap n times the value of V at $t+1$ (we shall assume that in state 1 nV is greater than K , while in state two is smaller, to make the manager exercise his option only in one state of the nature⁵). The asset's payoff becomes

$$V_{i,t+1} + \text{Max}(nV_{i,t+1}-K,0) \text{ for } i=1, 2.$$

In state 1 we have

$$V_{1,t+1} + (nV_{1,t+1}-K)$$

while in state 2 the payoff is

$$V_{2,t+1}$$

Given that the payoff in state 2 is the same, for the sake of the comparison we can leave it aside and concentrate on the payoff in state 1

Under the traditional method of valuation, the value of the project including the expansion options would be

$$V_{t,A} = \frac{\sum p_i * (V_{i,t+1} + \text{Max}(nV_{i,t+1} - K,0))}{1+k}$$

alternatively

$$V_{t,A} = [p * (V_{1,t+1} + nV_{i,t+1} - K) + (1-p) * V_{2,t+1}] \frac{1}{1+k}$$

rearranging terms

$$\frac{p}{1+k} * (V_{1,t+1} + nV_{i,t+1} - K) + \frac{(1-p)}{1+k} V_{2,t+1}$$

and

$$\frac{p}{1+k} * V_{1,t+1} + \frac{p}{1+k} * (nV_{i,t+1} - K) + \frac{(1-p)}{1+k} V_{2,t+1}$$

making use of what we know about the value V_t , we notice that the structure of value is equal to the original value of the business plus the expansion option

$$\frac{p}{1+k} * V_{1,t+1} + \frac{(1-p)}{1+k} V_{2,t+1} + \frac{p}{1+k} * (nV_{i,t+1} - K)$$

$$V_{t,A} + \frac{p}{1+k} * (nV_{i,t+1} - K)$$

⁵ Otherwise would not be an option given it is exercised anyway.

On the other hand, by using the risk neutral or contingent claim valuation method derived previously, we would have

$$V_{t,B} = \sum \tilde{p}_i * (Vi,t+1 + Max(nV_{i,t+1} - K)) \frac{1}{(1+r)}$$

which for the case of two states of the nature is given by

$$V_{t,B} = [\tilde{p} * (V_{1,t+1} + nV_{1,t+1} - K) + (1 - \tilde{p}) * V_{2,t+1}] \frac{1}{(1+r)}$$

following the same procedure of rearrangements of terms we have

$$V_{t,B} = \frac{\tilde{p}}{(1+r)} * V_{1,t+1} + \frac{\tilde{p}}{(1+r)} * (nV_{1,t+1} - K) + \frac{(1 - \tilde{p})}{(1+r)} * V_{2,t+1}$$

$$V_{t,B} = \frac{\tilde{p}}{(1+r)} * (V_{1,t+1} + nV_{1,t+1} - K) + \frac{(1 - \tilde{p})}{(1+r)} * V_{2,t+1}$$

which according to our initial results can be written as

$$V_{t,B} = \frac{\tilde{p}}{(1+r)} * V_{1,t+1} + \frac{(1 - \tilde{p})}{(1+r)} * V_{2,t+1} + \frac{\tilde{p}}{(1+r)} * (nV_{1,t+1} - K)$$

We observe that again the value of the business is equal to the original value plus the growth or expansion option

$$V_{t,B} = V_t + \frac{\tilde{P}}{(1+r)} * (nV_{1,t+1} - K)$$

comparing values for business obtained by each method, and simplifying for those terms equal in both derivations, we are left with the following simplified formula for traditional or DCF valuation

$$\frac{P}{(1+k)} * (nV_{1,t+1} - K)$$

while the corresponding for risk neutral valuation is

$$\frac{\tilde{P}}{(1+r)} * (nV_{1,t+1} - K)$$

Given that the second factor of the multiplication is the same for both, we can drop it off for comparison purposes and concentrate on the first. If a univocal relationship is established between both, we are done.

To this purpose, we make use of the components of any risk adjusted discount rate coefficient (1+k). It is formed by the risk free factor (1+r) plus a risk premia (1+θ)

$$(1+k)=(1+r)*(1+\theta)$$

Now we are allowed to make the last simplification. The risk free coefficient is present in both terms, so it can be dropped, then the comparison becomes

$$p/(1+\theta) \text{ vs } \tilde{p}$$

or rearranging

$$p \text{ vs } \tilde{p} * (1+\theta)$$

If the first term is greater, it would mean that valuation of growth options by traditional DCF method overestimates the true value of the expansion opportunity.

To prove this we use the basic axiom of the probabilistic theory, which says "the probability is a non negative number non greater than 1". Given that there is nothing in our derivation that can violate the axiom (the synthetic probability distribution comes out from a redistribution of mass at each point), and assuming the risk premia θ is positive (being a parameter we can take it for given), \tilde{p} can never be greater than p (if it was the case, and provided that we do not specify a specific value for this probability, we can always choose a value for \tilde{p} to get a p greater than one, which in turn violates the axiom, so the relation must hold for every p and \tilde{p}).

Hence, the first term is always greater than the second, and the traditional method of valuation overestimates the true value of the growth option.

III.3 Extension of the analysis from two states to n states of the nature

Having demonstrated the existence of overvaluation for the simple case of two states of the nature, we extend the framework to n states of the nature, where the random behaviour of the variable is assumed to follow

a binomial distribution with probability of success (upward movement) p , and n states of the nature. The maximum value that V can reach will have a probability of p^n associated, while the probability associated with the lowest value will be $(1-p)^n$

For any value of V which requires j upward movements out of n possible, the probability associated will be

$$B(n;j;p)=C_j^n * p^j * (1-p)^{n-j}$$

where B denotes the binomial distribution.

Under the risk neutral valuation, the set of values V can adopt does not change, only does the density associated to each value, so that the mean of the distribution is modified, adjusting it to the risk free return. As we saw, both methods give the same valuation for the underlying variable. The probability distribution thus obtained is much useful to value the options embedded in the project. We have to multiply each option payoff by its corresponding risk neutral probability, to obtain its expected, and then discount it to the risk free rate, obtaining the correct expected value. If we assume growth options are exercised when things go well, and we know that the true probabilities are greater for these states than their risk neutral counterpart, their complement for low value states will be smaller⁶, hence the inequality is reversed for low state values of the project. The demonstration is given by

take the upper bound, so that $j= n$, the true probability of this state or value would be

$$B(n;j;p)=C_n^n * p^n * (1-p)^{n-n} = p^n$$

while the risk neutral would be

⁶ Otherwise they will not add up to one.

$$B(n;n;p^{\sim})=C_n^n * p^{\sim n} * (1-p^{\sim})^{n-n} = p^{\sim n}$$

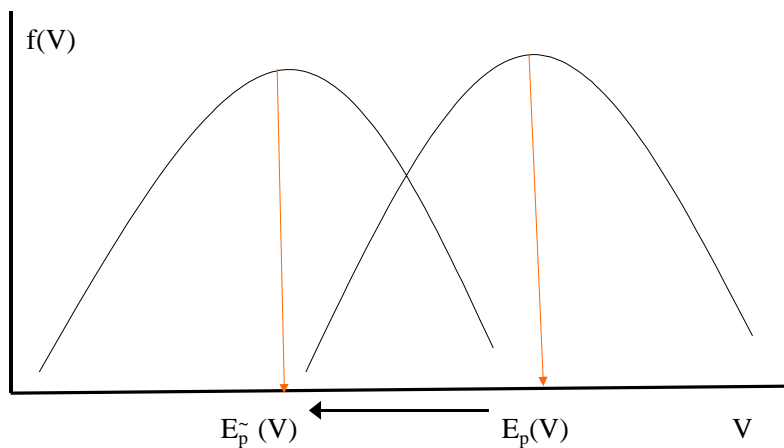
knowing that p^{\sim} is smaller than p , any increasing monotonic transformation has to respect the inequality, so it can be said the following inequality holds

$$\text{If } p \geq p^{\sim}, \quad \text{then } p^n \geq p^{\sim n}$$

Both probability distributions have to integrate to one, so the excess in the upper side has to be offset by a diminution on the value of probabilities for low values of the underlying variable, so the inequality is reversed for such values

$$p \geq p^{\sim}, \text{ then } (1-p)^n \leq (1-p^{\sim})^n$$

When extending the framework to a continuous distribution, where the binomial approximates the normal distribution as $n \rightarrow \infty$, a graph can be of much help



It can be observed there is a redistribution of mass in the probability distribution to change and reduce the first moment of the random variable (move the risk adjusted rate of return to the risk free, which is lower by assumed risk aversion). It is clearly seen that for high values of V the mass associated is lower under the risk neutral distribution, hence if the real distribution is used to value option it would be overvaluing its true value. This intuition confirms our previous derivations. In the same tense, for low value of V the mass associated is lower, but this change does not affect the value of the option, which has positive value only for high realizations of V (otherwise is zero, never negative).

IV Results

Due to this, though the valuation for the underlying asset is the same under both mechanisms, when it comes to evaluate growth options (horizontal, vertical or within the same market) embedded in the project, the traditional DCF overvalues the true option value. Although the discounting rate is smaller, and hence the discount coefficient is greater, which leads to increase the value of the option calculated by risk neutral valuation, this effect is not sufficient to offset the decrease in expected value due to the application of the new probability distribution.

If a project with embedded growth options is evaluated using traditional DCF, and possible values of V used to calculate the expected value of the project include the results of options already exercised, the result will be an overvaluation of the true value of the project.

Consider for instance a start up project. If for valuation purposes (to obtain values for different scenarios) the value of the company at one scenario is assumed to be in its mature stage (where the value at this

stage includes exercised growth options), then there would be a tendency to overstate the true value of the start up. The degree of overvaluation will depend upon the values adopted by the following parameters: r (risk free rate), k (risk adjusted rate), p (probability of high values for the project), V_u (the value of the project in a good state) and V_d (the value of the project if things do not go too well).

V Comparative Statics

A simulation model can provide more insights. Assume the two possible values the company can take are 135 in one scenario (with probability 43%) and 95 in the other (with probability 57%). The risk adjusted discount rate is assumed to be 10%. Under the traditional DCF methodology, the value of the project would be 100. Now assume that at the following period the company is able to expand further by paying a cost of 200 to obtain an expected value of two times the value of the company at $t+1$. This growth opportunity will be exercised only if the market proves to be good for the company (scenario 1). Then the following results are obtained

For the purposes of comparative static we change one parameter at a moment, keeping the others constant.

In the following table, we can observe the results of our changes in the values of the parameters. First we change the upper value of V , then the lower value of V , we continue by changing the risk free rate and the risk adjusted rate of return, and finally we change the value of the true probability p .

Initial value of parameters

V_u	130
V_d	95
r	5%
k	10%
p	43%
Expansion's Pay off	$2 * V_i$
Investment cost	K
Net payoff	$\text{Max}(2 * V_i - K, 0)$

	Initial Values	Increase V_u to 140	Decrease V_d to 85	Increase risk free rate r to 7%	Increase risk adjusted rate k to 12%	Increase probability p to 50%
Present value of the asset	100	103.9	94.8	100	98.2	102.3
Risk neutral probability p	29%	31%	32%	34%	23%	35%
Growth option value under traditional DCF	23.4	31.2	23.4	23.4	23.0	27.3
Growth option value under risk neutral valuation	16.3	23.9	18.5	19.2	13.3	20.2
Extent of overvaluation	44%	31%	27%	22%	73%	35%

- an increase on the upper possible value V_u reduces the excess of overvaluation
- a decrease on the lower possible value V_d reduces the extent of overvaluation
- an increase on the risk free rate r reduces the excess of overvaluation
- an increase on the risk adjusted discount rate k increases the excess of overvaluation
- finally, an increase on the real probability p of upward movements reduces the degree of overvaluation

Now we shall explain the intuition underlying these effects but first a note on how risk neutral probabilities are obtained is useful to include.

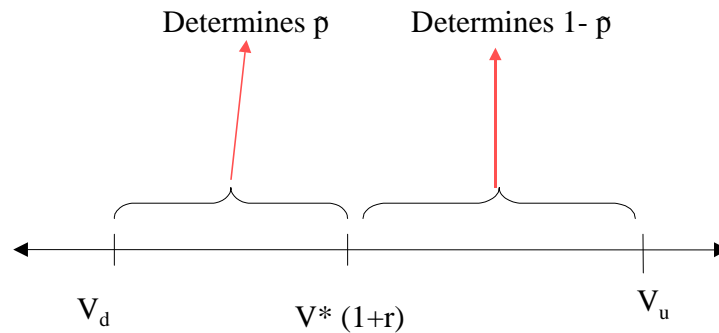
The probability \tilde{p} comes from

$$\tilde{p} = \frac{V^*(1+r) - V_2}{V_1 - V_2}$$

where V is

$$V = (p * V_u + (1 - p) * V_d) * \frac{1}{1 + k}$$

This can be better appreciated with the help of the following graph



An increase on the upper value V_u increases the expected value of the underlying asset. Given the methodology of calculation of the risk neutral probability \tilde{p} , we would expect the probability to diminish, however, this effect is more than neutralized by the move in the expected value of the asset (used together with the risk free rate of return to determine the risk neutral probabilities), which moves the division line between probabilities to the right. This effect overcomes the other, hence increasing \tilde{p} . This situation drives the risk neutral probability closer to

its real counterpart (which is assumed to be constant here), reducing the extent of overvaluation.

The decrease on V_d leads to the same effect. The changes on this extreme value are exactly the opposite as those described previously (the upper value going up is equivalent to the lower going down). In both cases the expected value of the underlying asset is affected, though in a contrarian sense, impacting on the divisory line between risk neutral probabilities. An increase on V_u or a decrease on V_d broadens the range between the extreme values, affecting in an opposite way the expected value of the underlying asset but affecting in the same way the risk neutral probability, bringing it closer to the real counterpart, therefore reducing the degree of overvaluation.

Both an upward movement on the risk free rate r , or a reduction on the risk adjusted rate k , can be synthesized in a change on the risk premia of the asset (the risk adjusted rate can be decomposed into two components, the risk free component and the risk premia).

Risk adjusted rate (k) = Risk free rate (r) + risk premia

An increase on r (keeping k constant) as well as a decrease on k (given r), can be assimilated to a decrease on the equilibrium risk premia. However, the effects on the dependent values are not the exactly the same⁷.

An increase on r does not change the expected value of the asset, but affects the line dividing the risk neutral probabilities. Given how this probability \tilde{p} is calculated, the division line is moved to the right, increasing it. This drives the risk neutral probability closer to the real probability, reducing therefore the extent of overvaluation.

The effect of an increase on k affects the expected value of the underlying asset moving the division line to the left, thereby reducing the

⁷ In fact the effects are the opposite.

risk neutral probability \tilde{p} and broadening the gap between the synthetic and the real probability.

Finally, an increase on p increases the expected value of the underlying asset. This moves the division line to the right, therefore increasing \tilde{p} and reducing the degree of overvaluation.

It comes out from these explanations that the analysis mainly passes through the study of the movements of the division line that makes up the values of the risk neutral probabilities \tilde{p} y $1-\tilde{p}$. It is not much complicated from a visual inspection to find out the results as a consequence of movements on the value of the parameters

VI Methodology

To the purpose of solving the problem of overvaluation detected and exposed previously, the following methodology is proposed to correctly evaluate the growth opportunities

- separate the outcomes of contingent decisions from the current value of the company.
- analyze the random structure of events the company faces.
- define a variation range for the possible values of the business, without including results of options.
- calculate the present value using the DCF method, to determine the value of the underlying asset, and with this in hand, determine the risk neutral probability distribution.
- use these probabilities to value the options, discounting the expected value to the risk free rate.

- finally, add the value thus determined to the value of the company.

We know it is not easy task, and that we have worked with a simplified model. However, the fact of thinking about contingent situations and possible outcomes represents a great advance to the company and manager's strategic thinking.

VII Applications and conclusions

Even though we know the results obtained must have been subject of extense research in the literature of financial options, now growing literature on real options is taking advance of the results previously obtained. It starts to be thought that options are everywhere within the company, and given that flexibility has value, this is the appropriate method to capture it. Throughout this paper it has been demonstrated that embedded growth options valued through traditional DCF have an overvaluation problem.

The intention of this paper was to show that valuation of projects and business with growth opportunities must take into account the overvaluation effect they are exposed to. The present value of a business is composed of two elements: the present value of assets in place and the growth opportunities.

Full Value = Value of assets in place + Value of growth options⁸

The weight of each component will be affected by the industry and the firm's own characteristics. To the extent that the company is into a

⁸ Myers S. 1977, "Determinants of Corporate Borrowing ". Journal of Financial Economics 5

mature industry, and the possibility of growing has been fully exploited and reflected in the current value of the company's assets, the growth component will tend to be relatively not significant with respect to the full value. On the other hand, for companies and industries in expansion or newly created industries, the most of the value will be captured by growth options, weighting more significantly towards the full value. This contingent growth will have associated a high volatility, due primarily to the uncertainty surrounding the market, the product or service, competitors and substitutes. Being more significant the option component for this kind of industries, the use of the traditional DCF model for valuation purposes will offer more problems, prompting overvaluation.

The most significant and illustrative example can be captured by the impact a tool like Internet has on growth opportunities for companies and industries. This development affects industries in not a symmetric fashion and to different extents. For those companies that are affected the most (needless to say that ecommerce companies are in this set), Internet creates a complete new world of opportunities, and also creates risk of overvaluing business due to the problems described, under the assumption that investors use the DCF model as a valuation tool. Options must be valued as their nature claims.

However, it was said throughout this paper that both methods are complements rather than substitutes. Risk neutral probabilities cannot be obtained without figuring out the current value of the underlying asset, for which DFC is appropriate. So they work together towards the same goal. Though each method has to be applied for the right situation to a proper analysis of the allocation of resources.

Our results are derived based upon a set of assumptions, so results are conditioned and the model developed is not much complicated. However, these assumptions are not far more restrictive than those involved in the derivations of models like the Capital Asset Pricing Model or the Black Scholes formula. Nevertheless, this fact should not stop us from relaxing assumptions and searching for results. This is a fantastic topic for future research.

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