Continuous-Time Option Games Part 2: Oligopoly and War of Attrition under Uncertainty

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Presentation Outline

- Oligopoly under uncertainty: the Grenadier's model
- War of attrition under uncertainty: oil exploration
- Changing the game approach: war of attrition enters as input to bargaining game
- Conclusions

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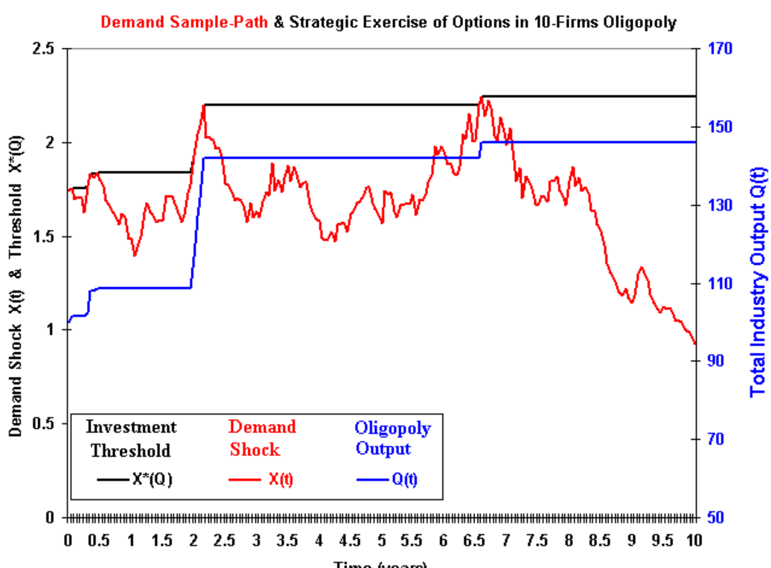
Oligopoly under Uncertainty

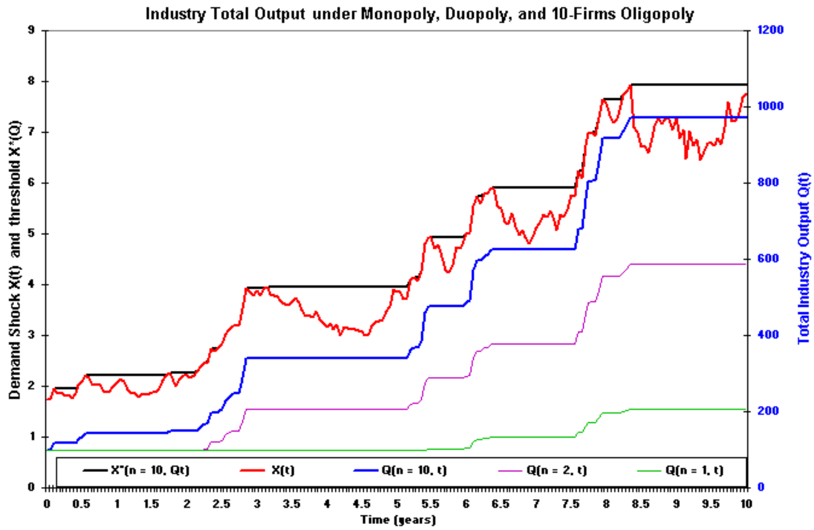
- Consider an oligopolistic industry with n equal firms
  - Each firm holds compound perpetual American call options to expand the production.
  - The output price P(t) is given by a demand curve D[X(t), Q(t)]. Demand follows a geometric Brownian motion.
- This Grenadier's model has two main contributions to the literature:
  - Extension of the Leahy's "principle of optimality of myopic behavior" to oligopoly (myopic firm ignoring the competition makes optimal decision);
  - The determination of oligopoly exercise strategies using an "artificial" perfectly competitive industry with a *modified demand function*.
    - Both insights simplify the option-game solution because "*the exercise game can be solved as a single agent's optimization problem*" and we can apply the usual real options tools, avoiding complex equilibrium analysis.
- We will see some simulations with this model in order to compare the oligopolies with few firms (n = 2) and many firms (n = 10) in term of investment/industry output levels
  - We will see the maximum oligopoly price, an *upper reflecting barrier*

Oligopoly under Uncertainty

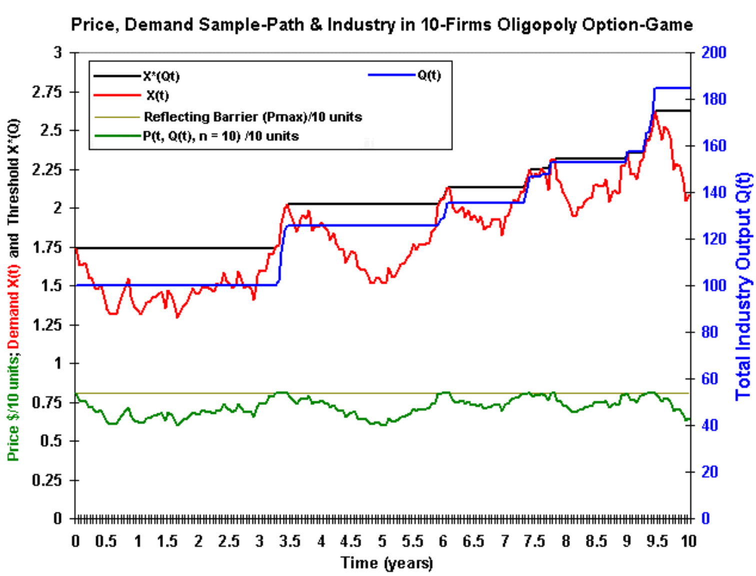


Jump to War of Attrition

Oligopoly under Uncertainty



Oligopoly under Uncertainty



War of Attrition under Uncertainty

- **Two oil firms i and j have neighboring exploratory prospects in the same geologic play (correlated prospects)**
  - **If the firm i drills the wildcat and find oil, the good news is public information and increases the firm j *chance factor* (CF_j) to find oil.**
    - **In case of negative information revelation, it reduces CF_j**
  - **So, there is an additional incentive to wait: we can *free-rider* the drilling outcome information from the neighboring prospect.**
    - **Waiting can be optimal even at the cost to delay the exercise of a deep-in-the-money exploratory option. It enlarges the waiting premium.**
  - **This *waiting game* is named *war of attrition*. The strategies are option exercise times ("*stopping times*") choices.**
    - **The firm drilling first is named the *leader* (L), stopping at t_L, whereas the firm choosing $t_F > t_L$ is named the *follower* (F). Here F > L.**
  - **This game is *finite lived*: there is a *common* legal expiration for the exploratory options (E) at t = T. These options depends on the oil price (P), which follows a geometric Brownian motion.**

War of Attrition: Compound Options

- **There are *compound options*: the exploratory option E(P, t; CF) with exercise price equal the drilling cost I_W, and, in case of exploratory success, the oilfield development option R(P, t)**
  - **The exercise of R(P, t) earns the oilfield NPV, which is function of the oil price, the reserve volume (B), the reserve quality (q), and the development investment I_D. Assume the following *parametric* equation for the develoment NPV ("business model"): $NPV = q\,B\,P - I_D$**
  - **Using the *contingent claims* approach, R(P, t) is the solution of:**

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 R}{\partial P^2} + (r-\delta)\,P\,\frac{\partial R}{\partial P} - r\,R + \frac{\partial R}{\partial t} = 0$$

- If $P = 0$, $R(0, t) = 0$
- If $t = T$, $R(P, T) = \max(q\,B\,P - I_D, 0)$
- If $P = P^*$, $R(P^*, t) = q\,B\,P^* - I_D$
- If $P = P^*$, $\dfrac{\partial R(P^*, t)}{\partial P} = q\,B$

Where: r = risk-free interest rate,
σ = oil volatility,
δ = oil convenience yield
P* is the optimal development threshold.

War of Attrition: The Exploratory Option

- If we find oil reserves when exercising the exploratory option E(P, t; CF) at the threshold P**, we will develop in sequence because P** > P* for all t < T.
  - By exercising E(P, t; CF) we get the *expected monetary value* (EMV), given by EMV = $- I_W$ + [CF . NPV] = $- I_W + [CF (q\ B\ P - I_D)]$
- Using the contingent claims method, E(P, t; CF) is the solution of:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 E}{\partial P^2} + (r - \delta) P \frac{\partial E}{\partial P} - r E + \frac{\partial E}{\partial t} = 0$$

- If P = 0, $E(0, t) = 0$
- If t = T, $E(P, T) = \max[- I_W + CF (q\ B\ P - I_D), 0]$
- If P = P**, $E(P^{**}, t) = - I_W + CF (q\ B\ P^{**} - I_D)$
- If P = P**, $\frac{\partial E(P^{**}, t)}{\partial P} = CF\ q\ B$

War of Attrition: Information Revelation Measure

- If firm j drills as leader, the follower firm i updates the chance factor CF_i upward to CF_i^+ with probability CF_j and downward to CF_i^- with probability $(1 - CF_j)$
- The degree of correlation between the prospects is given by the convenient learning measure named *expected variance reduction* ($EVR_{i\,|\,j}$), in %. Using probabilistic laws, the updated CF_i's are:

$$CF_i^+ = CF_i + \sqrt{\frac{1 - CF_j}{CF_j}} \sqrt{CF_i (1 - CF_i) EVR_{i|j}}$$

$$CF_i^- = CF_i - \sqrt{\frac{CF_j}{1 - CF_j}} \sqrt{CF_i (1 - CF_i) EVR_{i|j}}$$

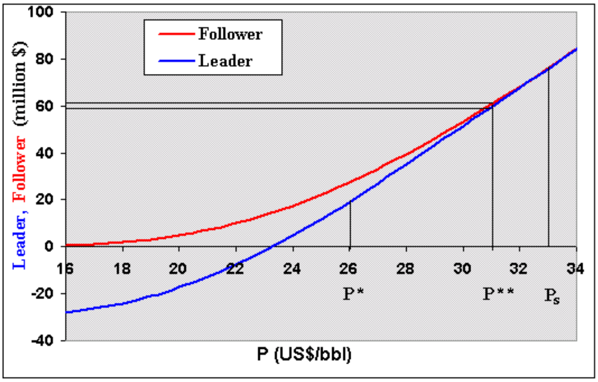
- The leader L(P, t) and follower F(P, t) values (for firm i) are:

$L_i(P, t) = - I_W + CF_i \,.\, R_i(P, t)$

$F_i(P, t) = CF_j \,.\, E_i(P, t; CF_i^+) + (1 - CF_j) \,.\, E_i(P, t; CF_i^-)$

War of Attrition: Thresholds and Values

- The relevant oil price thresholds are:
  - P*: the oilfield development option becomes "deep-in-the-money"
  - P**: the exploratory option becomes "deep-in-the-money"
  - P_F: the exploratory option threshold *after* the information revelation
  - P_S: upper limit for the strategic interaction (firm exercises its option regardless of the other player) $P_S(t) = \inf\{ P(t) > 0 \mid L(P, t) = F(P, t)\}$



Values for P = 31 $/bbl (in million $):

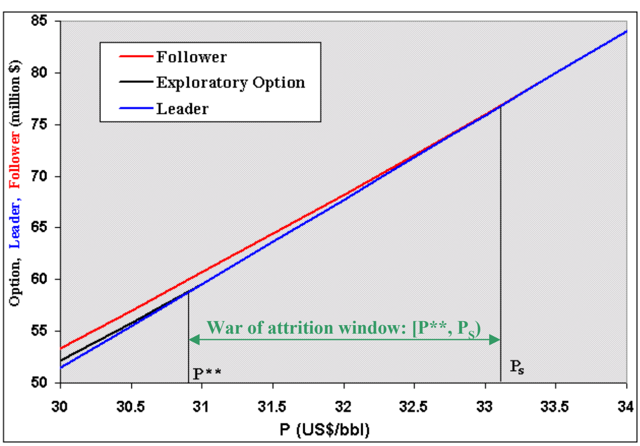
		Firm B: Wait	Firm B: Invest
Firm A	Wait	59.3 , 59.3	follower leader 60.7 , 59.6
	Invest	leader follower 59.6 , 60.7	Simultaneous 59.6 , 59.6

Convention: (shaded) = Nash Equilibrium

Inspired in Smit & Trigeorgis forthcoming book on option games

Interval Where the Game Matters

- The war of attrition game doesn't matter for prices below P** because the waiting policy is optimal anyway by options theory.
- For prices at or above P_S the game is also irrelevant because the follower value is equal to the leader value. The option exercise is optimal for *any* strategy of the other player.



In this example:

$[P^{**}, P_S) = [30.89, 33.12)$

War of Attrition Equilibria

- The leader will choose exercise at $t_L = \inf \{ t \mid P \geq P^{**} \}$, whereas the follower will do at $t_F = \inf \{ t \mid P \geq P_F \}$
- For the *symmetric* war of attrition there are two equilibria in pure strategies: firm i as the follower and firm j as the leader and vice-versa. In terms of stopping times: (t_{Fi}, t_{Lj}) and (t_{Li}, t_{Fj})
- For the *asymmetric* war of attrition, the unique pure strategy equilibrium is the *stronger* player as follower and the *weaker* as leader
  - The stronger player is the more patient and has higher threshold P_S.
  - Any small asymmetry is decisive to define this equilibrium
  - The paradoxical equilibrium with the weaker player being the follower *can* occur if we consider *disconnected sets* (multiple P_S regions). We don't consider it here.
  - Mixed equilibria: they are *ruled out* in the asymmetric war of attrition because the game is finite lived (and there is a payoff discontinuity at T)
- Instead further equilibrium analysis, we consider a more interesting alternative that can dominate all the possible war of attrition outcomes

Changing the Game: Cooperative Bargaining

- We follow the Brandenburger & Nalebuff (1996) advice: "*changing the game is the essence of business strategy*"
  - We'll check out the alternative game in which oil firms can play a *bargaining game* with a binding contract jointing the two assets
  - The *union of assets* has value U, and each firm has a *working interest* (w_i, w_j). The bargainer values are $U_i = w_i\, U$ and $U_j = w_j\, U$
  - The three main bargaining game theory branches are:
    - (a) cooperative bargaining (mainly the Nash's bargaining solution);
    - (b) noncooperative bargaining (alternating offer and counteroffer);
    - (c) evolutionary bargaining theory
  - The noncooperative bargaining solution converges to the Nash's cooperative solution if we allow a small risk of breakdown after any rejection to an offer (see Osborne & Rubinstein, 1994, 15.4)
    - We adopt the axiomatic Nash's cooperative bargaining solution here
  - We´ll see a motivating simple example to illustrate the "changing the game" relevancy in the oil exploration application.

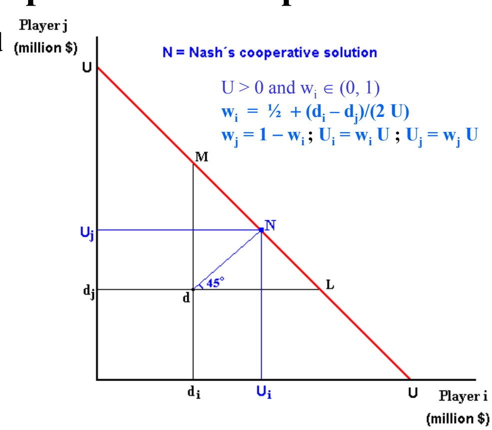
Jump example

Changing the Game: Motivating Example

- **Two oil firms have correlated neighbor prospects. Both are *expiring* and have negative *expected monetary value* (EMV)**
  - **$EMV_i = EMV_j = - I_W + [CF . NPV_{DP}] = - 30 + [0.3 \times 95] = - 1.5$ million $**
  - **In the noncooperative war of attrition context the value of each firm is zero because both will give up the wildcat tract rights.**
    - **Can we get anything better with a bargaining contract? Yes! Let us see how.**
  - **Partnership contract: the *union asset* U comprises these two prospects, $w_i = w_j = 50\%$, prospect j is drilled first and the second prospect i is optional (can be drilled in case of positive information revelation). So,**
  - **$U_i = U_j = 50\% \{EMV_j + [CF_j . Max(0, EMV_i^+)] + [(1 - CF_j) . Max(0, EMV_i^-)]\}$**
  - **Consider that the degree of correlation between the prospects is given by the expected variance reduction EVR = 10%.**
    - **With EVR = 10%, the updated chance factors for the second prospect i are $CF_i^+ = 52.14\%$ and $CF_i^- = 20.51\%$. Hence, $EMV_i^+ = 19.53$ and $EMV_i^- < 0$**
    - **Hence, the bargainer values are:**
      $U_i = U_j = 50\% \{- 1.5 + [0.3 \times 19.53] + [0.7 \times 0] \} = + 2.18$ million $
  - **So, the bargaining game permits to exploit all the surplus (information revelation + optionality), strictly dominating the noncooperative strategy**

The Cooperative Bargaining Nash's Solution

- **For the general case with $t \le T$, the union asset U values:**
  - **$U = Max\{ 0, EMV_j + [CF_j . E_i(P, t; CF_i^+)] + [(1 - CF_j) . E_i(P, t; CF_i^-)] \}$**
- **The cooperative bargaining game is defined by the pair (*S*, d), where *S* is the *feasible set* of bargaining outcomes and d is the *disagreement point*. The red line exploits all the surplus U.**
- **The Nash's solution is *unique* and is based in 4 axioms:**
  (1) invariance to linear transformations;
  (2) Pareto outcome efficiency (red line);
  (3) contraction independence; and
  (4) symmetry (45º).
- **The disagreement point (d) here is the equilibrium outcome from the war of attrition noncooperative game.**
  - **So, we combine war of attrition and bargaining games in this *changing the game* framework.**

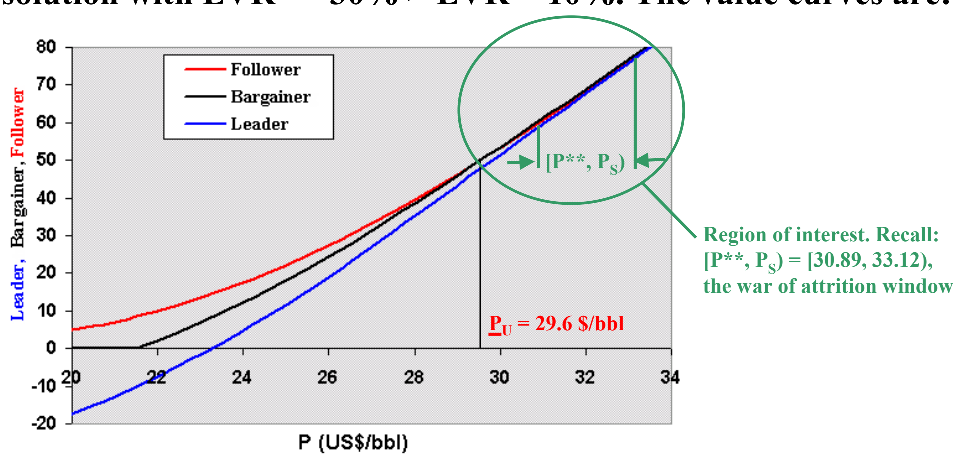


Cooperative Bargaining x War of Attrition

- **Denote EVR* the *expected variance reduction* given the *private* information provided by the partnership contract**
  - **The <u>public information</u> revelation obtained by the free rider follower in the war of attrition <u>is only a subset</u> of this private information.**
  - **So, in general EVR | private information > EVR | public information, or simply EVR* > EVR in order to compare the bargaining solution with the disagreement point (war of attrition) in fair basis.**
- **For the disagreement point we could place the equilibrium with the stronger firm being the follower and the other the leader. In the symmetric case it could be an average of F and L values.**
  - **However, we consider an extreme (and fictitious) case: $(d_i = F_i, d_j = F_j)$ both players with the highest war of attrition payoff (both follower!). Note that we <u>don´t</u> say that it is the best *bargaining* solution.**
  - **<u>If</u> in a state-space set (oil prices interval) the bargaining solution is better than this most favourable noncooperative alternative, we say that <u>bargaining game *dominates* the war of attrition in this set, for *any* war of attrition equilibrium</u> placed at the disagreement point.**

Cooperative Bargaining x War of Attrition

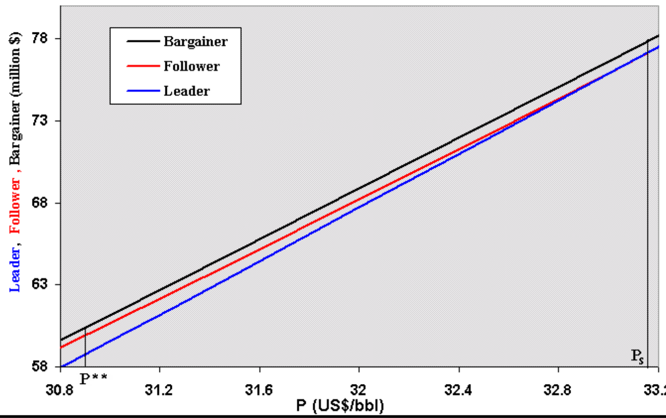
- **In the interval $(\underline{P}_U, \overline{P}_U)$ we guarantee that the bargaining game strictly dominates *any* war of attrition noncooperative equilibrium**
- **In the war of attrition example let us consider the Nash's bargaining solution with EVR* = 30% > EVR = 10%. The value curves are:**



Recall: the bargainer $U_i = w_i \,.\, \text{Max}\{\, 0, EMV_j + [CF_j \,.\, E_i(P, t; CF_i^+)] + [(1 - CF_j) \,.\, E_i(P, t; CF_i^-)]\, \}$

Cooperative Bargaining x War of Attrition

- The figure below is a zoom from the previous one in the interval where the war of attrition game matters: $[P^{**}, P_S) = [30.89, 33.12)$
  - See below the dominating bargainer value in this interval.
- In this example the bargaining game dominates the war of attrition game for the <u>entire interval</u> where the war of attrition matters, i.e. $[P^{**}, P_S) \subset (\underline{P}_U, \overline{P}_U) = (29.6, 36.7)$. So, change the game!



Conclusions

- The Grenadier's oligopoly under uncertainty adds up important methods to option-games toolkit, mainly:
  - Extension of the Leahy's "Principle of Optimality of Myopic Behavior"
  - Modified demand function to solve an "artificial" competitive industry
  - We saw simulations of prices and production for different oligopolies
- We analyzed a finite lived war of attrition game applied to oil exploration, with compound options and stochastic oil prices
  - The prize is the information revelation over the chance factor to find out oil
  - A small asymmetry combined with the game expiration can point out a unique perfect equilibrium (t_{Fi}, t_{Lj}) with the stronger player as follower
- We analyzed the possibility to change the game from the noncooperative war of attrition to cooperative bargaining game
  - We combine these games in a way that the war of attrition equilibrium enters as input (disagreement point) in the Nash's bargaining game.
  - The bargaining game *can* dominate any war of attrition outcome.
- Thank you very much for your time!