Evaluating Offshore Petroleum Leases Using Real Option Theory
- An application to the Central Mexican Gulf

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Bachelor's Thesis in Economics
Written by Leo Lundquist
Tutored by Professor Mats Persson

Stockholm University
Department of Economics

Abstract:

I examine whether a two-factor real option model, dependent on the spot price of crude oil and the convenience yield, can explain the highest bids in offshore petroleum auctions held by the U.S. Government. Applying the model to 23 viable offshore leases, offered in the Federal lease sale No. 178 part 1, suggests that the option model is successful in justifying the high bids. Sample mean option values are more than three times larger than the average high bids, which strongly contradicts the notion of a winner's curse in offshore petroleum auctions. Moreover, evidence is presented indicating disparities between the short- and long-term market price of convenience yield risk for crude oil futures on NYMEX.

1. INTRODUCTION

A growing body of empirical work suggests that ordinary investment evaluation techniques underestimate the value of ventures. Critics insist that discounted cash flow (DCF) analysis fails to account for the existence of flexibility in investment decisions [Hayes and Abernathy (1980); Laughton (1998)]. Real option models focus on describing uncertainty and in particular the presence of managerial options in investments. Concentrating on the many options available in projects develops investment theory by emphasizing the strategic aspects of the investment process. These real options might include the option to delay, expand, contract, or liquidate an investment, thus making it more valuable. Besides explicitly declaring value these models indicate how to best make use of options inherited in different projects.

Every year petroleum[1] companies bid hundreds of million Dollars for offshore petroleum leases, auctioned out by governments. Performing precise value assessments is very important for governments and bidding firms. Petroleum firms use their calculations as input to their bid decisions, whereas the U.S. Government performs cash flow valuations to establish a suitable presale reservation price. Paddock, Siegel, and Smith (1988) argue that government valuations have tended to underestimate high industry bids. Consequently raising the question whether neglecting the great flexibility in oil ventures leads to serious undervaluation of assets and missallocation of resources in the economy. As a result, substantial efforts should be spent in developing valuation models that can describe the coherent uncertainties of petroleum investment.

This thesis examines whether the highest bids in offshore petroleum auctions can be better explained by a real option approach than the government's DCF estimates. Given that neither method can explain the highest bids, is this evidence of overbidding in the federal lease auctions? Or could it be attributed to some other factor not captured by the option approach? I consider the option to delay exploration and development for 23 viable offshore leases in the Central Mexican Gulf using the same geological and cost data as the government. The vast majority of the oilfields were pure oil tracts, as a consequence all existing natural gas will be neglected. Sample tracts were auctioned out in the Federal lease sale No.178 part 1, which was held on March 27, 2001.[2]

The real option value of all tracts is estimated by using the model of Gibson and Schwartz (1990). Moreover, I also account for the exploration- and production lags. The valuation is performed by setting up a risk-free portfolio, which implies taking a position in the oilfield and two derivatives. Subsequently, a numerical method is used to estimate the option value, which is compared to the highest bid, the geometric mean bid, and the government's own estimate. More particularly, an explicit finite difference method[3] is used to solve the partial differential equation (P.D.E.) satisfying the value of the tracts.

The remainder of this thesis is organized as follows. Section 2 outlines relevant theory regarding petroleum, auctions, and real options. Section 3 contains a review of the literature on real options and petroleum assets. Section 4 presents the option valuation used in the comparison. Section 5 presents the empirical implementation of the model. Section 6 sums up and concludes.

[1] Petroleum is defined as oil and/or gas.

[2] The author would like to thank the U.S. Department of the Interior, Minerals Management Services (MMS) who have provided this thesis with information on all tracts used in the study. Although MMS has contributed with many useful comments any findings, conclusions, or recommendations expressed herein are those of the author.

[3] P. 419, Hull (2000).

2. THEORY

2.1 Oil Industry Characteristics

Crude oil is the world's most actively traded commodity and continues to be one of the most important natural resources used through out the world. Also, the petroleum industry is full of managerial flexibility due to the long shelf life of oil. Investment can thereby be delayed indefinitely as long as the oil is left for storage in the field. The light sweet crude oil futures contract traded on NYMEX is equivalent to the oil extracted in the Central Mexican Gulf and is quoted in Dollars per barrel.[4] This contract has become the world's most liquid for crude oil trading, as well as the world's largest volume contract on a physical commodity. The contract is used as an international pricing benchmark because of its' excellent liquidity and price transparency.[5]

Figure 1
Options Present in Offshore Oil Production[6]

The figure summarizes the many options available to an offshore leaseholder during the exploration and production stages in the petroleum industry. Each stage constitutes an option to complete the next stage.

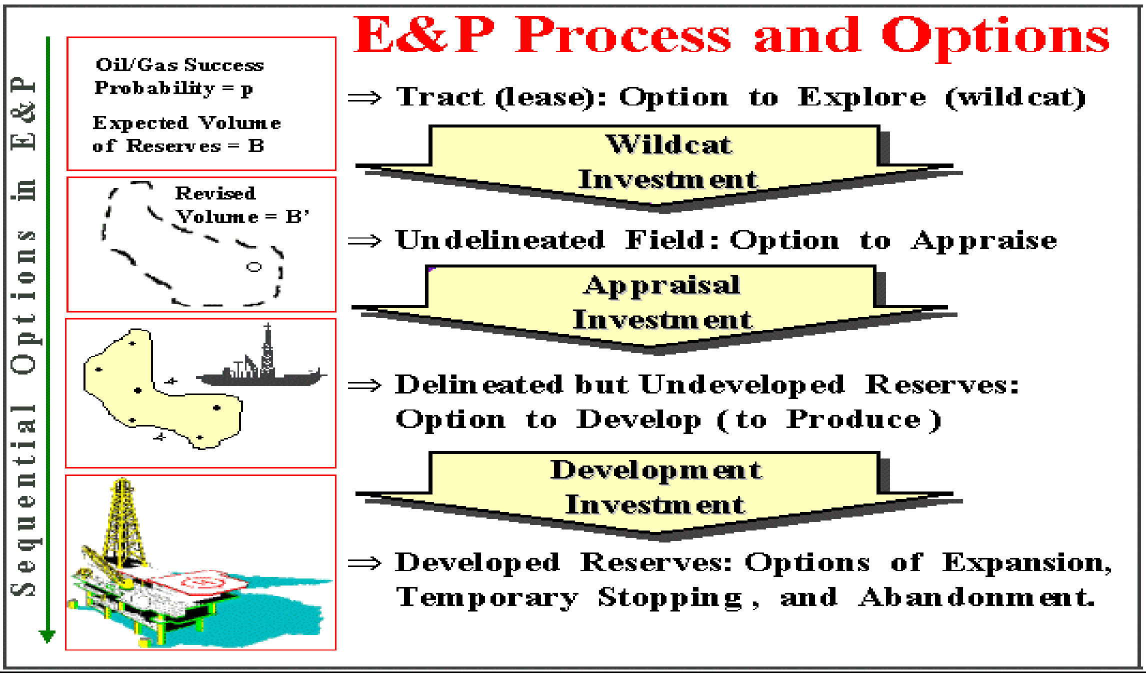

Fig. 1 provides an overview of the many options occurring to an offshore leaseholder before petroleum is obtained above the ground. Still, decisions to explore or develop may be more complicated in reality since leases are often held by several companies, who jointly decide when to explore or develop a tract. In addition, investment decisions can be influenced by the financial situation of the firm.[7]

The U.S. Department of the Interior, Minerals Management Service (MMS) handles U.S. Government lease auctions on petroleum assets. The auction is organized as a sealed-bid

[4] Equivalent to approximately 159 litres.
[5] www.nymex.com (2003-04-30).
[6] http://www.puc-rio.br/marco.ind/overview.html (2003-04-30).
[7] A more complete discussion on offshore petroleum leases is presented in Grafström and Lundquist (2002).

auction[8] where petroleum companies are able to bid either independently or as partnerships conditional on that they are authorized by the MMS. They are also forced to state the proportionate interest of each participating bidder. The MMS retains the right to withdraw any block prior to issuance of a written acceptance of a bid. The accepted bid will be the highest valid bid. To ensure that the government receives a fair return on the lease rights all high bids are evaluated in accordance with MMS bid sufficiency procedures.[9]

The lease term for a Mexican Gulf tract in deep water depths is either ten or eight years, and five years for shallow waters. All tracts have a minimum price and the leaseholder is obligated to pay a small annual rental rate until a large enough amount of petroleum is found. When production starts the lessee will instead pay a royalty rate.[10] The evaluation of the received bids is conducted by the MMS in two stages. In the first round the MMS accepts the highest bid for the fields considered being nonviable.[11] Bids on other tracts are accepted if there are at least three bids and the third bid is at least 50% of the highest qualified bid. These procedures are generally completed within three weeks of the bid opening. All other leases not fulfilling these requirements are further evaluated in the second phase.

Subsequently, MMS again evaluates whether there are tracts that are nonviable. The highest bids for such fields are generally accepted. Other tracts are evaluated according to whether they exceed the adjusted delay value (ADV) or the mean of the range of value (MROV). The MROV is a dollar measure of the government's estimate regarding the fields expected net present private value, given that the tract is leased in the current sale. The ADV is the minimum of the MROV and a calculation that assumes that the tract is sold in the next sale instead. The second phase bid adequacy determinations are normally completed sequentially over a period ranging between 21 and 90 days after the sale. The MMS can eliminate bids from the auction in case of any unusual bidding patterns. Upon acceptance, winners pay the remaining part of the bonus bid along with the first year's annual rental.

2.2 Petroleum Auctions

An auction can be described as a bidding process through which various goods are bought and sold. Choosing the appropriate auction format can be a fairly important decision, since it will affect the seller's revenue. A commonly used type is the traditional English auction,[12] where the seller requests gradually higher bids from potential buyers. Each bidder is aware of the current high bid during the entire auction. The winning bid will be the current high bid that no one is willing to surpass.

A sealed-bid auction differs from an English auction in that all bids are made at once in sealed envelopes. Consequently, all buyers are unaware of the others bids, which matter since the highest bid wins. However, the price paid by the winner will typically vary depending on whether it is a first-price auction or a second-price auction. In a first-price auction the highest bid equals the price, but in a second-price auction the price equals the second highest bid.

In a common-value auction the object to be auctioned out is roughly worth the same to all bidders. Nevertheless, buyers are unaware of the precise value and therefore bidder's estimates will vary. In the case of an offshore petroleum auction bidders will all know today's oil price but their estimates for the amount of petroleum reserves and the expected extraction

[8] To be further explained in section 2.2.

[9] For information on bid procedures see http://www.gomr.mms.gov/homepg/lsesale/lsesale.html (2003-04-30).

[10] Paid on each barrel of produced crude oil. There exists a minimum royalty fee for all leases and some of them also offer a royalty suspension up to certain volumes.

[11] Tracts which risk-weighted resource size is below minimum economic field size, for the relevant cost regime and anticipated future petroleum prices.

[12] Also referred to as an oral auction.

costs will typically differ. Hence, buyer's bids will tend to deviate.[13] Pindyck and Rubinfeld (2001) conclude that a first-price auction doesn't necessarily generate higher revenue than a second-price auction, since the buyers will change their strategies accordingly. So to pay as little as possible in case of success they will bid less in a first-price auction. The best strategy in a first-price sealed-bid auction would be to choose a bid that will be equal to or slightly above the reservation price of the buyer with the second-highest reservation price. It's never worth paying more than the second-highest reservation price. Consequently, auction theory concludes that first- and second-price auctions generate the same expected revenue. However, the obtained revenues can differ in practice between the two types of auctions, since bidders are unaware of the reservation prices of their peers.

Bids are not necessarily internal valuations of bidders or reservation prices, rather they are the outcome of a strategic bidding process. Provided that each buyer bids up to his reservation price the winner is likely to be the person with the largest positive error regarding the value of the product. This is known as the winner's curse, suggesting that the winner of a common-value auction is too optimistic and as a result overvalues the item. Lohrenz and Dougherty (1983) argue that winning bids in offshore lease auctions appear to systematically exceed true expected underlying tract values. Kagel and Levin (1986) also maintain that high bids tend to exceed true expected underlying values in controlled bidding environments.

In the case of an offshore oil tract with unknown reserve size and cost of extraction the bidders need to take the winner's curse into account. Otherwise they risk overpaying for the petroleum reserves. So companies should not only estimate their reserve value, but also acknowledge that all bids may be subject to error. Reducing your highest bid by an amount equal to the expected error of the winning bid is one way to handle the winner's curse.

Pindyck et al. (2001) claim that since petroleum firms have been bidding for reserves for several years they are able to estimate the expected error on the winning bid. Petroleum firms should therefore be able to take the winner's curse into account.

An English auction should be preferred to a sealed-bid auction from the perspective of a seller trying to maximize revenue. A buyer faces less uncertainty in an English auction, which reduces concerns about the winner's curse and thereby encourages more bidding. Indeed, this will mean that more risk-averse buyers dare to enter the bid process, since they can more easily account for the winner's curse and winning bids should thereby be higher.[14]

2.3 Real Option Theory

Many investments cannot be recovered and regularly involve great uncertainty concerning future prospects. These investments are defined as irreversible and are evaluated from a sunk cost perspective. Real options provide the opportunity but not the obligation to commit to a sunk cost. Thereby real options exhibit a claim on real assets. Projects inheriting options in regard to the investment decision are worth more than a similar project without flexibility. Consequently, advocates of real option models argue that the value of a firm can be seen as the sum of the net present value[15] (NPV) and the flexibility value.

The first step in performing a real option valuation involves choosing an appropriate stochastic process followed by the underlying asset. Valuing contingent claims[16] also requires assigning a risk-free rate of return to the underlying asset. Fortunately, the assumption of risk-neutrality is valid for both real options as well as financial options and thereby simplifies

[13] The bids are also likely to differ due to varying spot price forecasts among companies.
[14] MMS has actually proposed an English auction format but bidders rejected it for some reason.
[15] NPV refers to today's value of all future cash flows.
[16] Any investment right whose value depends on an underlying asset.

valuation of these derivatives. Table 1 provides an overview of the similarities between real and financial options.

Table 1
Analogy Between Real and Financial Options

The table reports the similarities between the parameters in the Black Scholes formula and a typical real option model. The resemblance makes it easier to understand and implement option methods on real assets.

Black Scholes Financial Options	*Real Options in Petroleum Investment*
Financial Option Value	Value of an Undeveloped Reserve (V)
Current Stock Price	Present Value of Developed Reserve (PV)[a]
Exercise Price of the Option	Investment Cost to Develop the Reserve (k)[b]
Stock Dividend Yield	Net Convenience Yield (δ)
Risk-Free Interest Rate	Risk-Free Interest Rate (r)
Stock Volatility	Volatility of Developed Reserve (σ)
Time to Expiration of the Option	Time to Expiration of Investment Right (τ)

[a] The value of the developed reserve is discounted for both the exploration lag and the production lag.
[b] Includes the exploration cost, annual rental rate, and discounted development cost.

Crude oil is primarily held for consumption reasons rather than investment. The convenience yield measures the advantages of owning a physical commodity, not obtained by holding a futures contract. Among them is the possibility of benefiting from supply shortages and demand increases. The commonly used arbitrage argument between futures and spot prices is summarized in Eq. (1) [Hull (2000)].[17]

$$f = Se^{(r-\delta)\tau} \tag{1}$$

where,

δ = net convenience yield
τ = time to maturity (T-t)
r = risk-free interest rate
S = spot rate

Evaluating derivatives on nontraded assets dependent on the convenience yield requires an estimate of the assets real growth rate and the market price of risk.[18] Subsequently, any derivative dependent on the convenience yield can be valued under a risk-neutral framework.

3. PREVIOUS LITERATURE

Paddock et al. (1988) evaluate offshore petroleum leases in the Mexican Gulf by examining how well different investment measures explain auction bids. The authors argue that historic government valuations have tended to underestimate industry bids. Using a one-factor real option model for 21 selected offshore tracts provides values closer to winning bids in comparison with the DCF estimates of the government.[19] Results also imply that reserves with low investment costs are likely to be developed earlier. However, these findings are dependent on the price assumption for the natural gas in the field. Still, the winning bids were

[17] Most real option models use a net convenience yield, the convenience yield less the storage costs.
[18] See Grafström et al. (2002) for a more thorough outline on stochastic processes, risk-neutrality, the convenience yield, and the market price of risk.
[19] Valuing both the exploration and development stage.

typically more than double the option value. A likely explanation would suggest that the winning firm has a more optimistic view on geologic circumstances and future spot prices.

Helfat (1989) compares auction bids for tracts in the Mexican Gulf before and after the oil embargo of 1973-1974. Pre-embargo and post-embargo prices are then evaluated using a portfolio choice model incorporating the impact of firm internal investment covariance on offshore tract prices. The author argues that the firm should be concerned with the total risk of all investments. Hence, the decision of how much to pay for a tract can be affected by other offshore investments. Findings indicate that a decrease in actual lease prices following the embargo[20] may to some extent reflect overbidding in the early 1970s rather than underbidding later in the decade.

Grafström et al. (2002) compare the real option value of an undeveloped North Sea oilfield with the DCF value. They use the Gibson and Schwartz model in comparison with a certainty equivalent value and a risk-adjusted cash flow value. Results imply that the option premium can range from 20-1000%, for reasonable spot prices. However, the premium over the risk-adjusted method can sometimes be insignificant since the valuation will be dependent on the spot price forecasts of managers. Still, the option criterion indicates an investment-timing rule that differs strongly from the risk-adjusted method. Moreover, the authors conclude that a substantial premium exists for pessimistic spot price predictions.

Furthermore, they summarize previous findings in real option studies by declaring that a vast majority find evidence of an option premium. In addition, one-factor models predicting mean reversion in crude oil spot prices are incapable of describing futures prices as are the models assuming that spot prices follow a geometric Brownian motion (GBM). Oppositely, multi-factor models among them the model suggested by Gibson et al. (1990) perform excellently in forecasting the long-run term structure of futures. It assumes that the convenience yield is mean reverting and that the crude oil spot price follows a GBM.[21]

4. METHODOLOGY AND DATA

4.1 Sample Selection

The public bid reading for federal lease sale 178 Gulf of Mexico region took place on March 28, 2001 in the Sheraton New Orleans Hotel, New Orleans, Louisiana. Overall the sale resulted in 547 tracts receiving 780 bids amounting to approximately $663 million.[22] 90 petroleum companies participated in the bidding. Bids were signed separately and sent sealed prior to 10 a.m. March 27, 2001.[23] Table 2 gives a brief summary of lease sale 178.

Only viable tracts in the second phase can be part of the sample since these are the only leases thoroughly economically evaluated by the MMS. All tracts receiving less than two bids are disregarded, since a single bid can't be presumed to represent industry valuations. Subsequently, the remaining 150 tracts are discriminated based upon which are considered to be nonviable in the first phase of the evaluation procedure of the MMS. These are all removed from the sample, now consisting of 111 lease offerings. Finally, all leases determined nonviable in the second phase of evaluation are removed, which leaves 40 tracts. Out of these 40 only 23 were thought to have recoverable oil assets, the rest were pure natural gas fields.

[20] Considering the higher oil prices after the embargo suggests that lease value should have been higher in this period.
[21] For a comprehensive summary on GBM, mean reversion, and multi-factor models see Grafström et al. (2002).
[22] http://www.gomr.mms.gov/homepg/lsesale/histstat.html (2003-04-01).
[23] If the bids are received later than specified above they will be returned unopened to bidders. Also, bidders may not modify or withdraw their bids unless they send a written withdrawal prior to the time specified above.

Table 2
Lease Sale 178 part 1 Statistics

The left column summarizes statistics on bids in the first phase of the auction. Whereas the right column indicates the total number of multiple bid tracts evaluated in the second phase.

First Phase		*Second Phase*	
Total tracts offered	4390	Multiple bid tracts	111
Tracts receiving bids	547	Viable multiple bid tracts	40
Total number of bids	780	Accepted bids for viable tracts	34
Multiple bid tracts	150	Rejected bids for viable tracts	6

Table 3 provides an overview of the final sample from which 17 pure gas tracts have been removed. Rejected bids in the second phase are included since neglecting them would bias the sample towards indicating overbidding. Only one of the sample tracts has confirmed amounts of petroleum and the rest are therefore defined as wildcat prospects. The sample consists of leases with relinquishment requirements varying between five-year, eight-year, and ten-year periods. The five-year tracts are subject to a higher royalty rate than the other lease contracts. In addition, all ten-year tracts in the sample are eligible for royalty suspension. In other words, firms are not obliged to pay royalty up to a certain amount of petroleum production.

Table 3
Final Sample Statistics

The table summarizes all 23 sample leases based on whether MMS accepted the high bid or rejected it. Tracts are discriminated based upon maturity length and whether they are pure oil tracts or not.

	Bid Accepted	*Bid Rejected*
Total sample tracts	18	5
Oil tracts	11	3
Oil and gas tracts	7	2
Five-year maturity	3	2
Eight-year maturity	-	2
Ten-year maturity	15	1

Leases eligible for royalty suspension[a]: 16

[a] The tracts not eligible for royalty suspension are the eight-year and five-year maturity leases

All winners are awarded the contracts no later than 90 days after the auction. All contracts are therefore assumed to be awarded on June 27, 2001, which will be used as the start date of the valuation. Table 4 lists some general tract information along with some financial parameters.

Table 4
General Lease Characteristics and Financial Parameters

The table reports general tract characteristics and parameters used in the option valuation. As emphasized previously any presence of natural gas will be neglected. All interest rate data has been provided by Reuters.

Parameter	*Data*
Relinquishment requirement	5, 8, or 10 years
Corporate tax rate	35%
U.S. ten-year Government Bond 2001-06-27[a]	5.11%
Royalty rates	12.5% or 16.67%
Forecasted field life[b]	8-16 years

[a] Recalculated to a continuous interest rate.
[b] Refers to the life of the field once production has begun and naturally differs between tracts.

All tract-specific data has been provided by the MMS and includes the following items for each offshore lease:[24]

1) Quantities of recoverable oil, condensate, and gas reserves
2) Geological probability that the tract is dry
3) Expected exploration cost for both dry and wet tracts
4) Expected delineation cost
5) Expected development cost
6) Total estimated operating cost over the life of the field
7) Annual depreciation
8) Exploration and production lag
9) Expected field life

Total operational costs for each field are divided into equally sized annuities for each year of expected field life. Extracted reserves are treated similarly and are assumed to be certain. Furthermore, the quality of the oil reserves will not be taken into account, rather all reserves are presumed to be sold at market prices. Depreciation is tax deductible and is therefore multiplied by the corporate tax rate and added back to cash flows.[25] Changes in operating working capital are disregarded. All sample ten-year leases are subject to royalty suspension up to a certain amount of produced crude oil.

The exploration cost, delineation cost, annual rental rate, and the development cost are added together as a lump sum investment expenditure.[26] Expected exploration costs for each tract are estimated by multiplying the geological probability of success with the expected exploration cost of a wet tract and subsequently adding the corresponding expected cost for a dry tract. The total development cost reported by the MMS is discounted to the day of lease allocation. Paddock et al. (1988) argue that exploration lags are often negligible, so production and exploration lags will also be treated as a lump sum. The lag consists of the time between the beginning of field exploration and the extraction of petroleum. As a consequence, the option model will value the compound option of exploring a wildcat tract and immediately developing it. The underlying presumption is that it is optimal to begin development immediately after exploration. With no geological uncertainty, collapsing together the development and exploration option is always appropriate [Paddock et al. (1988)]. The real option estimates are then compared with ADV estimates of the MMS. Both sets are also compared with the highest industry bids and the geometric mean bid on the same tracts, considering the contrary views in section 2.2 on whether high bids represent industry valuations or not.[27]

4.2 Real Option Valuation

Considering the long time horizon of a real option valuation emphasizes the importance of closely monitoring the evolution of variables over time. Conclusions of how the selected state variables affect derivative value can be drawn by carefully studying time series data. A growing body of empirical work argues that one-factor models assuming mean reversion or geometric Brownian motions in spot prices perform poorly in predicting the term structure of oil futures [Schwartz (1997a)]. Conversely, two- and three-factor models taking convenience

[24] The author is unable to reveal any tract-specific data, since it is confidential and therefore cannot be disclosed.
[25] Depreciation is also divided into equally sized annuities for the entire depreciation period.
[26] I use undiscounted expected exploration costs because drilling lags in the sample are rather short.
[27] The geometric mean is used since the distribution of industry bids is so strikingly skewed and thereby the geometric mean becomes a better measure of central tendency.

yield variability into account have proven to be very successful in pricing futures. Consequently, this study will apply the two-factor model developed by Gibson et al. (1990) and tested by Schwartz (1997a) in the valuation of the option to delay investment.[28]

Utilizing the futures valuation formula of the two-factor model enables appraisal of prices for light sweet crude contracts of all maturities.[29] So for any given day a spot price *S* and an annualized convenience yield *δ*, implies a certain term structure of futures prices. Hence, the futures formula is used to estimate the term structure of futures prices on Jun.27, 2001 for the entire life of each tract, including the corresponding production lag.[30] Subsequently, each tract is evaluated as if the decision to invest is already taken. The predicted term structure for the corresponding tract life is then used to compute the certainty equivalent cash flows, which are discounted with the risk-free interest rate.[31] Consequently, the certainty equivalent (CEQ) value of the developed lease is expressed as[32]

$$NPV_{CEQ} = -k + \sum_{t=T}^{N} e^{-rt}\left[\left(\left(1-Y_t\right)Q_t F\left(S,\delta,T\right) - C_t\right)(1-h_t) + h_t D_t\right] \quad \textbf{(2)}$$

$$PV_{CEQ} = \sum_{t=T}^{N} e^{-rt}\left[\left(\left(1-Y_t\right)Q_t F\left(S,\delta,T\right) - C_t\right)(1-h_t) + h_t D_t\right] \quad \textbf{(3)}$$

where,

k = initial investment
N = life of the oilfield once production has begun
r = continuous yield on a ten-year government bond
Y_t *= royalty rate year t*
F(S, δ,T)= predicted futures price
Q_t *= number of barrels of oil to be extracted in year t*
C_t *= total cost of production year t*
h_t *= corporate tax rate year t*
D_t *= planned depreciation year t*

Schwartz (1997b) illustrates that taking a position in two derivatives and the offshore lease results in a partial differential equation (P.D.E.) satisfied by the value of the option to invest V(S,δ,τ),[33] which means adding Eq. (2) to the P.D.E. describing the value change of any crude oil derivative dependent on the spot rate and the convenience yield.[34]

$$\frac{\partial V}{\partial S}(r-\delta)S + \frac{\partial V}{\partial \delta}(\eta(\alpha-\delta) - \lambda\sigma_\delta) + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma_S^2 S^2 + \frac{1}{2}\frac{\partial^2 V}{\partial \delta^2}\sigma_\delta^2 + \frac{\partial^2 V}{\partial S \partial \delta}\sigma_S \sigma_\delta \rho\, S - \frac{\partial V}{\partial \tau} - rV$$
$$-k + \sum_{t=T}^{N} e^{-rt}\left[\left(\left(1-Y_t\right)Q_t F\left(S,\delta,T\right) - C_t\right)(1-h_t) + h_t D_t\right] = 0 \quad \textbf{(4)}$$

[28] See Grafström et al. (2002) for a complete summary on the two-factor model and the assumed stochastic processes of the convenience yield and the spot rate. Moreover, Eq. (10) and Eq. (11) in the same study show how to express the risk-adjusted and risk-free growth rate of any two-factor crude oil derivative.
[29] The futures valuation formula is outlined in Eq. (14) in Grafström et al. (2002).
[30] Remember that there also is an exploration lag, which is included in the production lag for each tract in this study.
[31] All costs are assumed to be predictable as mentioned previously.
[32] In contrast to Grafström et al. (2002) this study incorporates a royalty rate in the option valuation and disregards the exchange rate.
[33] Equivalent to the value of the oilfield.
[34] The P.D.E. describing the value change of the derivative is summarized in Eq. (11) in Grafström et al. (2002).

where,

δ = annualized net convenience yield
η = annualized reversion rate in the convenience yield
α = mean convenience yield
λ = market price of convenience yield risk
ρ = correlation between process residuals
σ_S = standard deviation of the spot rate
σ_δ = standard deviation of the annualized convenience yield
τ = time to maturity (T-t)

All five corresponding boundary conditions to Eq. (4) are equivalent to the boundary conditions used in Grafström et al. (2002). Unfortunately, Eq. (4) is missing a closed form analytical solution satisfying the option to invest. As a result, the P.D.E. is solved numerically using an explicit finite difference method for each specific tract.[35] Remembering that the option to delay is an American option necessitates the implementation of a backward iteration at each discrete point in time in the numerical solution. The iteration is carried out to see when investment is optimal, for all possible values of the convenience yield and the spot rate.

4.3 Estimation of Real Option Parameters

All model parameters will be estimated over a historic period of eight years, since the offshore tracts will provide the leaseholder with cash flows over at least an eight-year period. As in Grafström et al. (2002) the spot price is approximated by the closest maturity futures contract traded on the NYMEX.[36] The implied annualized convenience yield is then found by rearranging Eq. (1) for weekly futures data over the period Jun.26, 1992 - Jun.22, 2000.[37]

In order to solve Eq. (4) parameters η, α, and σ_δ are estimated between Jun.1992-Jun.2000 using the seemingly unrelated regression (S.U.R.) summarized in Table 6, following Grafström et al. (2002).[38] The correlation between the process residuals is generated by the S.U.R, whereas the standard deviation of the crude spot price is found by analyzing time series data for the eight-year estimation period.

Finally, the assessment of the exogenous market price of convenience yield risk, λ, is performed accordingly. I use prices of crude oil contracts traded on the NYMEX and compare these to the estimates generated by the valuation model. The optimal value of the market price of risk is obtained by minimizing the sum of squared errors (SSE) for the chosen contracts by varying λ.[39] For a total of 838 weekly futures observations, the corresponding annualized convenience yield and interest rate are used to compute theoretical prices for each week. The estimation of λ is optimized by using futures with distant maturities, since accurate pricing of long time maturity contracts is undoubtedly most important for the valuation of long run petroleum cash flows.[40] Seven annual contracts with maturities from one to seven years are used in the optimization. The U.S. ten-year Government Bond is used in the pricing of the six- and seven-year contracts, the one-year U.S. T-bill is used for the one-year contract, and the five-year U.S. Government Bond is used for the price estimation of the four remaining

[35] The numerical solution is programmed in Matlab and is summarized in the Appendix.
[36] All futures data is provided by NYMEX, where 35 monthly light sweet crude futures contracts are traded.
[37] The implied convenience yield is computed for the last trading day of each week during the entire period. See Grafström et al. (2002) for a more comprehensive explanation of the convenience yield and spot rate estimation.
[38] Weekly data of the spot rate and convenience yield are jointly regressed against their lagged values.
[39] More precisely, I use the analytical solution in Eq. (14) presented in Grafström et al. (2002).
[40] Although, some of these contracts are traded under poor liquidity they are vital in estimating long-run crude prices, which are critical in petroleum investment.

contracts. Most previous studies have used short-term contracts in the parameter estimation and implicitly presumed that estimates should be adequate also in pricing contracts of longer maturity. To assess whether the market price of risk is dramatically different using short-term contracts I also use futures with maturities ranging from two to six months.[41]

Findings presented by Gibson et al. (1990) indicate that the two-factor model's assumption of a constant market price of risk is unsuitable for such a highly volatile parameter. Nevertheless, accurate pricing can still be attained despite the unrealistic assumption. So the out of sample pricing ability is tested on futures between Nov.2000 - Jun.2001,[42] in order to verify whether the optimized λ is sensible. The mean pricing error (MPE) and the root mean square error (RMSE) are calculated for each contract in order to determine whether there exists a general overpricing and the magnitude of it. The errors are then compared to the residuals of Gibson et al. (1990), Schwartz (1997a), and Grafström et al. (2002).

5. EMPIRICAL ANALYSIS

5.1 Real Option Parameters

Since the real option value is dependent on so many parameters it is necessary to closely examine the distribution of the variables. To control whether the random walk[43] and lognormal distribution assumption of crude prices is supported by the data I analyze the OLS regression results in Table 5. Each weekly logarithmic spot return is regressed against the lagged value.

Table 5
Time Series Properties of ln(St/St-1)

The table reports the regression parameters using weekly price notations between Jun.1992-Jun.2000.

Period	b	$t(b)$	DW^a	R^2	σ_S^b	N^c
Jun. 92- Jun. 00	-0.0060	-0.12	2.00	0.00	30.85%	418

OLS model: $\ln(S_t/S_{t-1}) = a + b\ln(S_{t-1}/S_{t-2}) + \varepsilon_t$

[a] DW denotes the Durbin-Watson statistic.
[b] σ_S denotes the annualized standard deviation of $\ln(S_t/S_{t-1})$ over the period.
[c] N denotes the number of observations.

The resulting regression parameters in Table 5 support the notion of spot prices following a random walk. The drift term is not significant during the estimation period. Analyzing the explanatory power of the regression leads to the conclusion that historic returns are unable to explain future returns. In addition, studying the Durbin-Watson statistic implies no presence of positive autocorrelation between residuals, so regression estimates are fairly accurate. Examining Fig. 2 lends further support for the random walk assumption and there seems to be no indication of strong mean reversion in spot prices.

Table 6 provides strong evidence of high and significant mean reversion in the convenience yield, agreeing with previous studies suggesting strong mean reversion [Gibson et al. (1990); Bessembinder, Coughenour, Seguin, and Monroe Smoller (1995)]. Any deviation from the convenience yield mean of 10.95% was adjusted within approximately

[41] The three-month U.S. T-bill is used for pricing futures with maturity of two to four months. Other short-term contracts are valued using the six-month U.S. T-bill.
[42] Again using Eq. (14) in Grafström et al. (2002).
[43] Equivalent to a GBM.

eight weeks during the eight-year estimation period. Also, the volatility of the convenience yield is extremely high as is the positive correlation with spot prices, agreeing with previous research.[44] Fig. 2 gives further support to the notion of a stochastic mean reverting convenience yield.

Figure 2
Annualized Convenience Yield and the Spot Price of Light Sweet Crude in Dollars

The figure displays the evolution of the Light Sweet Crude spot price, approximated by the nearest maturity contract on the NYMEX, and the annualized convenience yield calculated using Eq. (1) for weekly futures prices between Jan.1992-Jul.2001.

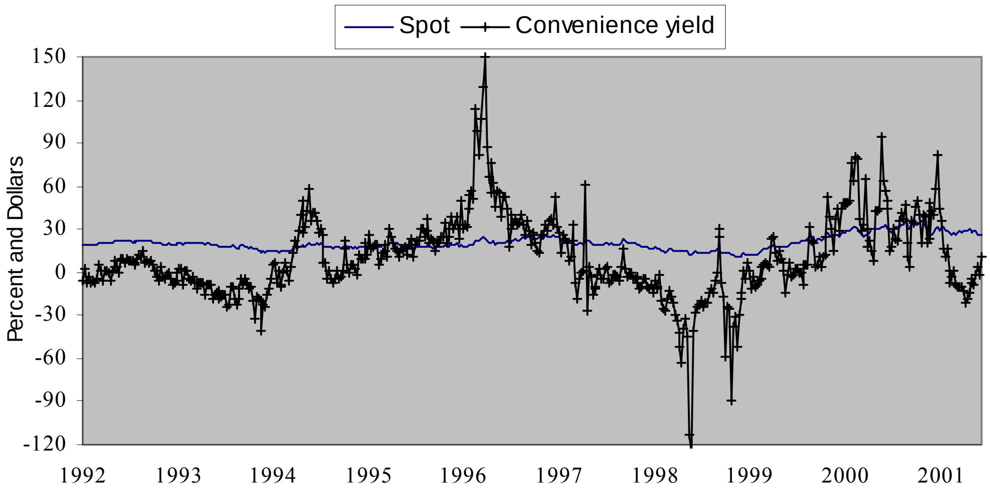

Table 6
Estimation of the Parameters of the Joint Stochastic Process[a]
Followed by Δδ and $\ln(S_t/S_{t-1})$

The table reports the parameters of the seemingly unrelated regression performed using weekly price notations between Jun.1992-Jun.2000.The logarithmic spot price change and the change in the convenience yield were regressed jointly against their lagged values using the below equations.

Period	η^b	$t(\eta)$	α	$t(\alpha)$	σ_δ^b	$\rho_{S,\delta}$	R^2	N^c
Jan. 90-Dec. 99	6.6934	5.08	0.1095	1.81	93.06%	0.55	0.06	418

$$\ln(S_t/S_{t-1}) = a + b\ln(S_{t-1}/S_{t-2}) + \varepsilon_t$$
$$\Delta\delta_t = \beta_0 + \beta_1\,\delta_{t-1} + e_t$$

[a] The seemingly unrelated regression model was fitted to estimate the coefficients of $\Delta\delta_t$ and $\ln(S_{t-1}/S_{t-2})$ jointly regressing, respectively, the former variable δ_{t-1} and the latter on its lagged value.
[b] The estimates of η and σ_δ have been annualized.
[c] N denotes the number of observations.

The optimized λ of 0.2489 in Table 7 was estimated from 838 weekly futures contracts, with maturities from one to seven years over the eight-year estimation period. Moreover, the optimized market price of convenience yield risk led to a within sample MPE of $-0.25 and to an RMSE of $2.64. In addition, the short-term market price of risk is presented.

There clearly exists a difference in λ in regard to the timeframe. It is evident from Table 7 that valuation of futures with less time to maturity is more accurate. However, long-run futures data is less frequent, which makes valuation difficult. In addition, a big part of the SSE for the long-run futures is due to just a few observations, which tend to neutralize each

[44] As expected by the theory of storage, arguing that when the oil supply is scarce benefits of owning oil are high as is the spot price.

other according to the MPE. Apparently the valuation of distant maturity petroleum derivatives is extremely troublesome in times of very high or low crude oil prices.

Table 7
Summary Statistics on Pricing Errors in Optimizing the Market Price of Risk, λ

The table presents the optimized value of the market price of risk. Estimation of the optimal λ for long- and short-run contracts was performed by minimizing SSE over an eight-year period.

Period	λ	*MPE* [a]	*RMSE* [b]	*SSE* [c]	*N*
Long maturity futures[d]	0.2489	-0.25	2.64	5840.61	838
Short maturity futures[d]	0.0512	-0.06	0.52	563.43	2090
Period: Jun.1992-Jun.2000					

[a] MPE refers to the mean pricing error in Dollars. Where N denotes number of observations, $\hat{F}$ the theoretical futures price, and F the actual futures price.

$$MPE = \frac{1}{N}\sum_{n=1}^{N}(\hat{F}_n - F_n)$$

[b] RMSE refers to the root mean square error in Dollars.

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\hat{F} - F)^2}$$

[c] SSE refers to the sum of square errors.
[d] Long maturity futures refers to the seven contracts ranging from one to seven years, while short maturity futures includes five contracts with lengths from two to six months.

Furthermore, Table 7 lends support to previous studies finding a positive market price of convenience yield risk for both short- and long-term futures, implying that investors demand a higher return for bearing convenience yield risk. However, economic theory indicates that the presence of convenience yield risk positively correlated with spot prices should decrease variability in futures contracts, thereby reducing the investors required return.[45] As a consequence, the risk premium of the convenience yield should be negative. Gibson et al. (1990) find evidence of a highly negative market price of risk for the period of Jan.1984-Nov.1988 for all available contracts. Oppositely, Schwartz (1997a) detects a positive market price of risk for the period of Jan.1985-Feb.1995, on the NYMEX as does Grafström et al. (2002) on the International Petroleum Exchange (IPE) in London. Consequently, the notion of a positive market price of risk is strengthened by findings from two exchanges, over three different estimation periods during the past two decades, and using two different parameter estimation methods. Overall, pricing accuracy is better in studies indicating a positive λ.

5.2 Out of Sample Pricing Performance

Since carrying out a real option valuation implies estimating long distant cash flows it is vital to test the performance in pricing out of sample futures contracts. Long maturity futures are valued between the period Nov.2000-Jun.2001 to detect any presence of measurement error in the parameter estimation. The selected contracts had maturities from one to seven years. Table 8 shows the worsening performance of the model for contracts with longer maturity. Gibson et al. (1990) and Grafström et al. (2002) make the same discovery.[46] One way to remedy would be to use weekly updates of λ, but real option models assume a constant

[45] One way to confirm this is to observe Eq. (1).
[46] One explanation would be the poor structure of the crude oil futures market resulting in decreasing liquidity and thereby less reliable price quotes for the long-term segment.

λ. Moreover, updating the market price of risk is more likely to improve valuation of short-term contracts than long-term, since λ is rather constant among longer contracts. The optimized λ renders a reasonable pricing performance for the model. Errors presented in Table 8 typically range between 7.5-15%. Indeed, this might seem on the high side but it should be emphasized that long distant cash flows are not critical to tract value. Rather it is the first few years that are important and still there is often a long production lag until petroleum can be extracted.[47]

Table 8
Summary of Pricing Errors in Valuing out of Sample Futures Contracts

The table summarizes the performance of the two-factor model in weekly pricing of futures contracts, for the period of Nov.17, 2000-Jun.15, 2001. The test includes contracts with up to seven years maturity. MPE and RMSE are expressed in U.S. Dollars.

Maturity (years)	*MPE*	*RMSE*	*SSE*	*N*
1	1.91	2.15	138.97	30
2	3.21	3.41	348.93	30
3	3.75	4.19	140.39	8
4	4.03	4.75	90.31	4
5	3.56	4.42	77.99	4
6	3.08	4.11	67.60	4
7	2.36	3.74	56.04	4
In Percentage of Mean Spot Price [a]				
1	6.58%	7.41%		
2	11.04%	11.74%		
3	12.60%	14.09%		
4	12.93%	15.24%		
5	11.41%	14.16%		
6	9.86%	13.18%		
7	7.57%	12.00%		

Period: Nov.2000 - Jun.2001

[a] Mean spot price, $29.04, is calculated as the arithmetical mean of the spot price for the period Nov. 2000-Jun.2001. However, the mean spot price corresponding to the three-year contract was $29.74 and even higher, $31.18, for longer futures.

Nonetheless, in comparison with the study performed by Schwartz (1997a) it is evident that the estimates are clearly less accurate. His results for the same maturity contracts range between 1.5-7.5%, clearly superior to this study. Several different explanations are plausible. The first one is the difference of parameter estimation methods: This study uses the parameter appraisal method of Gibson et al. (1990), while the Schwartz study uses the more complex Kalman Filtering. Another rationale could be that this study has fewer observations, which implies that a few outliers can affect the aggregate pricing ability substantially.[48] During the estimation period crude prices were high and the theoretical prices computed were inaccurate in predicting the term structure for long-run futures. However, removing the trading days containing spot prices over $30 improves pricing performance.[49] In addition, the model seems

[47] Moreover, on June 27, 2001 when all tracts are assumed to be awarded the pricing error for the one-year contract is only one cent and for the two-year contract approximately one Dollar.
[48] Contracts traded on NYMEX with maturities of four years and more all have expiration dates in December and therefore there only exists data for these maturities for a month each year.
[49] Five observations were removed for the one-year and two-year contract resulting in an RMSE of 5.82% and 10.41% respectively.

to do poorly for extremely low crude prices when the in-sample data is used in the optimization of the market price of risk. Nevertheless, for spot prices somewhere between $15-$25 per barrel term structure estimates are satisfying. Analyzing MPE values in Table 8 suggests a general overpricing of contracts, agreeing with previous studies.

5.3 Tract Value Analysis

The advantage of evaluating tracts with a real option approach lies in the avoidance of subjective projections about future spot prices. In addition, estimating real option value does not demand a forecast of a risk-adjusted discount rate, since a risk-free rate is used in the discounting of certainty equivalent cash flows. Table 9 summarizes the parameters used in the option valuation.

The option value for each tract is found by solving the P.D.E. in Eq. (4) for corresponding lease-specific data on Jun.27, 2001. More specifically, all leases are valued by predicting the term structure of futures prices and incorporating the value of delaying exploration and development, assuming a predetermined oil extraction rate and certain forecasted costs. Since underlying data is provided by the MMS, differences between the option valuation estimates and ADV estimates should be due primarily to differences in the financial valuation techniques. Although, analysts using the same geological information and cost data might end up with significantly different values.[50] Also, data used by the MMS may deviate from industry expectations for quantities of petroleum and development costs.

Table 9
Review of Parameters Used in the Real Option Valuation

The table reports the estimated parameters used in the real option valuation as expressed in Eq. (4). The parameters have previously been reported in Table 4, 5, 6, and 7.

S^a	δ^a	r^a	α	η	$\rho_{S,\delta}$	λ	σ_δ	σ_S
$25.61	-1.67%	5.11%	0.1095	6.6934	0.55	0.2489	93.06%	30.85%

[a] The spot price, convenience yield, and instantaneous risk-free interest rate are all collected on Jun. 27, 2001. The current rate of the ten-year government bond approximates the long run risk-free interest rate applied in the estimation of the term structure.

Table 10 presents simple correlation coefficients measuring the degree of linear association between the alternative measures of lease value. Comparing results in Table 10 to findings of Paddock et al. (1988) indicates a substantially higher correlation between MMS estimates and the winning bids, suggesting that MMS is either using a more efficient estimation method or that bidders have become more familiar with government estimation procedures. Another plausible explanation implies that the difference in assessment of geologic potential has diminished between the two over the past years. Also, the correlation between real option estimates and the top bids is twice as high, indicating that the option approach is successful in evaluating the unique risk profiles among sample tracts. On the other hand, the relation between the option value and the government estimates is lower in this study, but still evident. It could be a sign of the real option approach capturing some sort of investment value that the government overlooks or possibly an indication of some sort of measurement error.

Further comparing findings of this study with the results of Paddock et al. (1988) suggests that the assumption of spot prices following a GBM in the latter study provides an insufficient description of futures prices. However, some caution in comparing the two studies should be taken since most of the tracts in the Paddock study were gas tracts that were in the money,

[50] Due to different spot price forecasts and cash flow models.

and typically had low development costs. Furthermore, they had only a five-year lease term. Perhaps the option values of the latter study would have been relatively higher if ten-year leases also had been valued as in this study.

Table 10
Correlation Coefficients Between Valuations

The table reports correlation coefficients between the option value (OV), government ADV, geometric mean bid (GB), and the high bid (HB) for the total sample of 23 leases.

	OV	ADV	GB	HB
OV	1.00			
ADV	0.52	1.00		
GB	0.20	0.44	1.00	
HB	0.44	0.70	0.76	1.00

Table 11 presents estimates of average sample value using four different measures. Sample mean value using the real option approach amounts to $17.99 million, whereas the average value for the high bids is $5.36 million, and the average government estimate is only $1.70 million. Findings strongly disagree with the notion of a winner's curse present in offshore petroleum auctions. Sample values for the real option approach are approximately three times higher than the highest bids and ten times higher than government ADV estimates. Also, only three tracts in the sample received a high bid exceeding the real option value.

Another important aspect of the option approach for firms is the strategic aspect of when to explore and subsequently develop an acquired offshore lease. Real option models indicate when it is optimal for a firm to start exploration and development of a tract. For the sample used in this study results indicate that exploration should be delayed considering the high option values presented.

Table 11
Mean and Standard Deviations for Valuations

The table reports summary statistics for each valuation methodology and for the aggregate 23 tracts in the sample. OV-ADV etc. represents the tract-by-tract difference in OV and ADV valuations. All amounts are quoted in $ millions.

Valuation Methodology	Sample mean	Sample standard deviation	Standard error of the mean
OV	17.99	16.44	3.43
HB	5.36	7.51	1.57
GB	1.51	1.70	0.36
ADV	1.70	2.36	0.49
OV-HB	12.63	14.76	3.08
OV-GB	16.48	16.19	3.38
OV-ADV	16.29	15.34	3.20
ADV-GB	0.20	2.21	0.46
HB-ADV	3.66	6.09	1.27

Apparently option theory provides a strong explanation for the historic discrepancy between government and industry valuations in petroleum auctions. Consequently, results contradict the notion of a winner's curse. There seems to be substantial value in delaying exploration and development of tracts, a value not captured by the reservation prices of the MMS. In addition, leases in the sample with low investment costs and lower amount of reserves are likely to be developed earlier since the value of delaying investment is smaller. Results imply that the government should account for the option to delay, since any uncertainty about future spot prices can drastically increase bonus bids, especially for high-cost tracts. Hence, auctioning out tracts today should account for what happens to future high bids in case of rising crude prices.

Furthermore, Table 11 shows that the difference between ADV estimates and auction high bids is smaller than in the study by Paddock et al. (1988), which indicated a 400% difference in estimates. The decrease in disparity between the two was also implied by Table 10, suggesting an increase in correlation. Moreover, in contrast to previous studies the geometric mean bid is now smaller than the ADV. Again, this can be an indication of either an increased correspondence in government and industry assessment regarding geological potential or an increase in similarity of appraisal methods.

Nevertheless, the significant differences between option values and high bids raise the question whether option methods are credible in assessing tract value and whether the parameter estimation of this study has been carried out properly. Considering the geological uncertainty, spot price uncertainty, high investment costs, and the inherent managerial flexibility suggests that there undoubtedly exists value that a static cash flow valuation can't account for. Although, determining the exact size of this hidden value is difficult since geological and economical assumptions differ among firms. All sample tracts are out of the money and consequently all the tract value consists of the option to delay exploration. Clearly, the negative NPV values suggest that futures price predictions generated by the model are not overoptimistic. Rather, there should be cause for more disbelief regarding the estimations of the convenience yield and its' volatility.[51] Perhaps parameter estimates would have been different if they had been appraised from long-term contracts.

Using real option models implies estimating several volatile parameters.[52] Furthermore, it demands making assumptions that sometimes contradict reality. In particular, parameters are estimated from historical data not necessarily describing the future. Some of the parameters are in addition assumed constant over the life of the tract. Namely the risk-free interest rate, the volatility of the state variables, the correlation between state variables, and the market price of risk. Another important aspect is that security trading is assumed to be continuous and involve no transaction costs.

Also, real option models overlook some vital aspects of reality in investment decisions. First, petroleum ventures frequently involve great expenditures, increasing the possibility of financial distress for a single firm. Such costs are not included in option models and would naturally decrease lease value for any bidding firm. Although, these financial distress costs can be reduced by entering partnerships bidding for offshore leases.[53] By also taking into account that extraction rates are predetermined and that geological uncertainty is considered only in the calculation of exploration costs suggests that the assumption of risk neutrality is questionable.

Still, the two-factor model values only the crude oil present in the field, all natural gas has been disregarded, clearly a questionable assumption for some of the oil and gas tracts. Moreover, removing the tracts from the sample that received bids that the MMS rejected decreases the difference between the sample mean for the option estimates and the accepted high bids.[54]

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[51] These estimates could also affect the appraised mean reversion, since it is derived by carrying out a regression on convenience yield data.

[52] See Grafström et al. (2002) for a brief summary on the sensitivity of the two-factor model.

[53] Partnerships bidding for tracts is a quite common occurrence in offshore petroleum auctions in the Gulf.

[54] The sample mean high bids now correspond to 40% of sample mean option value and the correlation between the estimates rises to 0.61, compared to the previous figures of 30% and 0.44.

6. SUMMARY AND CONCLUSIONS

I examine whether a two-factor real option model, contingent on the spot price of crude oil and the convenience yield, is successful in explaining the highest bids in offshore petroleum auctions held by the U.S. Government. More specifically, the model values the option to delay exploration and development for 23 viable offshore tracts auctioned out in the Federal lease sale No.178 part 1. Subsequently, these values are compared with the high bids, geometric mean bids, and the government's own estimates of lease value. Using the same geological and cost data as the government suggests that the real option approach is successful in justifying the high bids. Findings display sample mean option values amounting to $17.99 million, compared with $5.36 million for the high bids, and $1.70 million for government estimates. This strongly contradicts previous studies documenting the notion of a winner's curse present in offshore petroleum auctions.

Moreover, the correlation between real option estimates and the high bids is twice as high as in previous studies, indicating that the option approach is successful in evaluating the unique risk profiles among sample tracts. Option theory clearly provides a strong explanation for the historic discrepancy in petroleum auctions between government and industry valuations. There seems to be substantial value in delaying exploration of tracts not captured by the reservation prices of the government. Results imply that the government should account for the option to delay, since any uncertainty about future spot prices can drastically increase bonus bids, especially for high-cost tracts.

Nevertheless, the significant differences between option values and high bids raise the question whether option methods are credible in assessing tract value and whether the parameter estimation of this study has been carried out properly. Since all sample tracts were out of the money it can be argued that the prediction of crude prices is unlikely to be overoptimistic. A more credible source of error would be the estimation of the convenience yield from short-term data, which could have been done from long-term contracts instead. Another cause for a large difference could stem from the fact that option models neglect the apparent costs of financial distress in the petroleum industry for a firm. The inherent risk in these huge investments can decrease lease value for any bidding firm in the real world.

However, this study has disregarded all present natural gas, clearly a disputable assumption for some of the oil and gas tracts. Moreover, removing leases with high bids rejected by the government from the sample decreases the difference between the sample mean for the option estimates and the accepted high bids.

Evidence suggesting differences between the short- and long-term market price of risk is also presented, which stresses the importance of estimating parameters from long-term contracts when valuing distant cash flows. Furthermore, the market price of convenience yield risk for futures with long and short maturities is proven to be positive. An economic anomaly detected previously by Schwartz (1997a) and Grafström et al. (2002).

An interesting extension to this study would be to include the abandonment option to further evaluate the option features of offshore leases. More research should also be carried out on tracts that are perceived to be economically nonviable by the government to see whether option features can imply different procedures for the government. Finally, an interesting study would be to examine whether a real option model is successful in describing the opening and closing decisions of petroleum firms. Are these decisions influenced by the fortunes of the firms other tracts or are they connected to specific managerial factors?

Appendix

The syntax in Matlab has been separated into three parts to create a simple overview.
Main: Uses the previously defined partial derivatives and performs the numerical solution.
Rhside: Summarizes the boundary conditions and defines the size of the steps.
NPV_Fun.m: $-k+\sum_T^N e^{-rt}\left[((1-Y_t)Q_t F(S,\delta,T)-C_t)(1-h_t)+h_t D_t\right]$
V_S_Fun.m, V_S_Fun.m, V_Sd_Fun.m, V_SS_Fun.m, V_dd_Fun.m, and **V_Fun** are all equivalent to the Grafström et al. (2002) study.
Main:
E=[];

% parameters	parameters=	% 39) : C_t11	0
% 1) : r	[0.0511	% 40) : C_t12	0
% 2) : D_t1	0	% 41) : C_t13	0
% 3) : D_t2	0	% 42) : C_t14	0
% 4) : D_t3	0	% 43) : C_t15	0
% 5) : D_t4	0	% 44) : C_t16	0
% 6) : D_t5	0	% 45) : C_t17	0
% 7) : D_t6	0	% 46) : C_t18	0
% 8) : D_t7	0	% 47) : C_t19	0
% 9) : D_t8	0	% 48) : C_t20	0
% 10) : D_t9	0	% 49) : Q_t1	0
% 11) : D_t10	0	% 50) : Q_t2	0
% 12) : D_t11	0	% 51) : Q_t3	0
% 13) : D_t12	0	% 52) : Q_t4	0
% 14) : D_t13	0	% 53) : Q_t5	0
% 15) : D_t14	0	% 54) : Q_t6	0
% 16) : D_t15	0	% 55) : Q_t7	0
% 17) : D_t16	0	% 56) : Q_t8	0
% 18) : D_t17	0	% 57) : Q_t9	0
% 19) : D_t18	0	% 58) : Q_t10	0
% 20) : D_t19	0	% 59) : Q_t11	0
% 21) : D_t20	0	% 60) : Q_t12	0
% 22) : alpha	0.1095	% 61) : Q_t13	0
% 23) : eta	6.6934	% 62) : Q_t14	0
% 24) : rho	0.55	% 63) : Q_t15	0
% 25) : lambda	0.2489	% 64) : Q_t16	0
% 26) : sigma_delta	0.9306	% 65) : Q_t17	0
% 27) : sigma_S	0.3085	% 66) : Q_t18	0
% 28) : k	0	% 67) : Q_t19	0
% 29) : C_t1	0	% 68) : Q_t20	0
% 30) : C_t2	0	% 69) : Y_t1	0
% 31) : C_t3	0	% 70) : Y_t2	0
% 32) : C_t4	0	% 71) : Y_t3	0
% 33) : C_t5	0	% 72) : Y_t4	0
% 34) : C_t6	0	% 73) : Y_t5	0
% 35) : C_t7	0	% 74) : Y_t6	0
% 36) : C_t8	0	% 75) : Y_t7	0
% 37) : C_t9	0	% 76) : Y_t8	0
% 38) : C_t10	0	% 77) : Y_t9	0

```
% 78) : Y_t10          0          % 85) : Y_t17     0
% 79) : Y_t11          0          % 86) : Y_t18     0
% 80) : Y_t12          0          % 87) : Y_t19     0
% 81) : Y_t13          0          % 88) : Y_t20     0
% 82) : Y_t14          0          % 89) : h_t       0.35
% 83) : Y_t15          0          % 90) : N         19
% 84) : Y_t16          0          % 91) : X_t       1];

for refin=5
for refint=1
% grid;
delta_min=-0.2;
delta_max=1;
S_max=45;
S_min=0;
N_delta=2^refin+1;
N_S=2^refin+1;
hdelta=(delta_max-delta_min)/(N_delta-1);
hS=(S_max-S_min)/(N_S-1);
dtau=0.001/refint;
[delta,S]=meshgrid(delta_min+hdelta:hdelta:delta_max-hdelta,S_min+hS:hS:S_max-hS);
delta((N_S-1)/4,(N_delta-1)/4);
f=zeros(size(S));
tau=0;
NPV=NPV_Fun(delta,S,parameters);
NPV=NPV.*(NPV>0);
for tau=0:dtau:10000*dtau/refint-dtau
  s1=dtau*feval('rhside',f,tau,hdelta,hS,delta,S,parameters);
  s2=dtau*feval('rhside',f+s1/2,tau+dtau/2,hdelta,hS,delta,S,parameters);
  s3=dtau*feval('rhside',f+s2/2,tau+dtau/2,hdelta,hS,delta,S,parameters);
  s4=dtau*feval('rhside',f+s3,tau+dtau,hdelta,hS,delta,S,parameters);
  f=f+(s1+2*s2+2*s3+s4)/6;
  f=f.*(f>0);
  f=f.*(NPV<f)+NPV.*(NPV>f);
  subplot(1,2,1)
  surf(delta,S,f);
  title(tau+dtau)
  drawnow
  subplot(1,2,2)
  surf(delta,S,NPV_Fun(delta,S,parameters));
  title(tau+dtau)
  drawnow
end
end
end
```

Rhside:

```
function dfdtau=rhside(f,t,hdelta,hS,delta,S,parameters);
k=parameters(28);
% expanding the solution matrix with the BC.
```

```
bc1=[0 S(1,:) 0]*0;   % S=S_min
bc2=S(:,1)*0;         % delta=delta_min
bc3=NPV_Fun(delta(1,end)+hdelta,S(:,1),parameters)+k;     % delta=delta_max
bc4=NPV_Fun([delta(1,1)-hdelta delta(1,:) delta(1,end)+hdelta],S(end)+hS,parameters)+k; %
S=S_max
f=[bc1;[bc2 f bc3];bc4];
%% f=V
V=f(2:end-1,2:end-1);
V_dd=(f(2:end-1,3:end)-2*V+f(2:end-1,1:end-2))/(hdelta^2);
V_SS=(f(3:end,2:end-1)-2*V+f(1:end-2,2:end-1))/(hS^2);
V_Sd=(f(3:end,3:end)-f(1:end-2,3:end)+f(1:end-2,1:end-2)-f(3:end,1:end-2))/(4*hS*hdelta);
V_d=(f(2:end-1,3:end)-f(2:end-1,1:end-2))/(2*hdelta);
V_S=(f(3:end,2:end-1)-f(1:end-2,2:end-1))/(2*hS);
dfdtau=V_SS*V_SS_Fun(t,delta,S,parameters)+...
      V_dd*V_dd_Fun(t,delta,S,parameters)+...
      V_Sd*V_Sd_Fun(t,delta,S,parameters)+...
      V_S*V_S_Fun(t,delta,S,parameters)+...
      V_d*V_d_Fun(t,delta,S,parameters)+....
      V*V_Fun(t,delta,S,parameters)+...
      F_Fun(t,delta,S,parameters);
```

NPV_Fun.m:

```
function F=NPV_Fun(delta,S,parameters);
k=parameters(28);
N=parameters(90);
r=parameters(1);
X_t=parameters(91);
h_t=parameters(89);
alpha=parameters(22);
lambda=parameters(25);
sigma_S=parameters(27);
eta=parameters(23);
sigma_delta=parameters(26);
rho=parameters(24);
t=(0:N);
alpha_hat=alpha-lambda*sigma_delta/eta;
NPV=0*S-k;
for n=1:length(t)
  T=t(n);
  A=(r-alpha_hat+sigma_delta.^2/eta^2/2-sigma_S.*sigma_delta*rho/eta)*T+...
  sigma_delta.^2/4*(1-exp(-2*eta*T))/eta^3+...
  (alpha_hat*eta+sigma_S.*sigma_delta*rho-sigma_delta.^2/eta)*(1-exp(-eta*T))/eta^2;
  D_t=parameters(1+n);
  C_t=parameters(28+n);
  Q_t=parameters(48+n);
  Y_t=parameters(68+n);
   NPV=NPV+exp(-r*T)*(((1-Y_t)*Q_t*S.*exp(-delta*(1-exp(-eta*T))/eta+A)-C_t*X_t)*(1-
h_t)+h_t*D_t);
end
F=NPV;
```

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