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Real Options Theory for Real Asset Portfolios: the Oil Exploration Case

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By: Marco Antonio Guimarães Dias

– Senior Consultant (internal) by Petrobras, Brazil

– Adjunct Professor of Finance (part-time) by PUC-Rio

Revised paper copy? Send e-mail to marcoagd@pobox.com

Presentation Outline

- Previous literature on portfolio of real options, correlation role and the paper focus cases.
- Learning model with two compound real options assets.
- Learning measures and correlation coefficient.
- Synergy model: sharing infrastructure investments.
- Portfolio value with learning & synergy: the now-or-never case.
- Portfolio value with learning & synergy with option to delay the options exercise.
- Portfolio value x correlation & the probabilistic learning limits (Fréchet-Hoeffding limits for the existence of bivariate distrib.)
- Charts on the effect of synergy & learning on portfolio value before and at the expiration.
- Concluding remarks.

Literature on Portfolio of Real Options

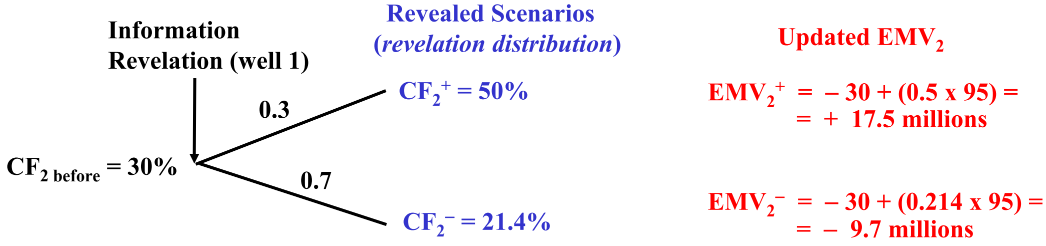
- **The portfolio of real options literature is still in its infancy.**
  - **This paper aims to contribute focusing two compound real options, considering synergy, learning and option to delay.**
- **Most of the previous cases focused real options on a *basket of mutually exclusive assets*, e.g., a portfolio of technologies to develop a project: option on the maximum of n risk assets.**
  - **In this case is very known that correlation (ρ) decreases the real options portfolio value and that the correlation signal matters.**
  - **Here we'll see that the contrary can occur: synergy value between real assets increases with ρ and learning option increases with ρ^2.**
    - **Childs et al. (1998), with other model, also found that learning ↑ with ρ^2.**
- **The application focus is *petroleum exploration and development assets*, but this theory can be easily adapted for R&D assets.**
  - **J.L. Smith (2004) also focus oil exploration real options portfolio, but his focus is the optimal stopping for a sequence of dry holes.**

The Basic Learning Model for Oil Exploration

- **Last year I presented a learning measure theory, showing that the *expected percentage of variance reduction* (η^2) has very nice mathematical properties and for Bernoulli distributions $\eta^2 = \rho^2$.**
  - **Chance factors in oil exploration (and R&D) are Bernoulli distr.**
- **The exploratory option exercise payoff, named Expected Monetary Value (EMV), is given by: $EMV = -I_W + CF \, . \, NPV_{dev}$**
  - **I_W = "wildcat" drilling investment; CF = chance factor to find out oil (success probability); NPV_{dev} = oilfield development NPV.**
- **Consider that NPV_{dev} is function of the oil prices (P), modeled with a *geometric Brownian motion* (GBM). Let this function be:**
  - **$NPV_{dev}(P) = q \, B \, P - I_D$, where q = reserve economic quality (∝ productivity); B = reserve volume; I_D = development investment.**
- **Consider two exploratory prospects sharing common geologic properties. In case of success in one prospect (oilfield discovery) it increases the chance factor in the other prospect (correlated).**

The Basic Learning Model for Oil Exploration

- Learning occurs in the prospect 2 chance factor (CF_2) with the information revelation from the prospect 1 exercise outcome.
  - Numerical example: two equal prospects with $CF_1 = CF_2 = 30\%$; $\rho = +28.6\%$; for both, I_W = \$ 30 millions; NPV_{dev} = \$ 95 millions
  - No learning: $EMV_1 = EMV_2 = -30 + (0.3 \times 95) = -\$\,1.5$ million
  - But with *sequential exercise* (w/ learning), the portfolio is valuable:



  - If drilling is instantaneous and if it is a now-or-never option:

$$\Pi = EMV_1 + [CF_1 \,.\, Max(0, EMV_2^{+})] + [(1 - CF_1) \,.\, Max(0, EMV_2^{-})] \Rightarrow$$
$$\Rightarrow \Pi = -1.5 + [(30\% \times 17.5) + (70\% \times 0)] = +\$\,3.75 \text{ millions}$$

  - So, *learning and optionality* make valuable this portfolio of prospects

The Basic Learning Model for Oil Exploration

- The equations for CF_2 after learning with prospect 1 outcome are ("+" is good news/prospect 1 success; "–" means bad news):

$$CF_2^{+} = CF_2 + \sqrt{\frac{1 - CF_1}{CF_1}}\,\sqrt{CF_2\,(1 - CF_2)}\;\rho$$

$$CF_2^{-} = CF_2 - \sqrt{\frac{CF_1}{1 - CF_1}}\,\sqrt{CF_2\,(1 - CF_2)}\;\rho$$

- In oil exploration *negative* correlation does not make sense. But in R&D there are applications where $\rho < 0$ has meaning.
  - For sake of generality, let us consider all range of correlations.
- There are learning limits, i.e., for a given CF_1 and CF_2 it is not possible any ρ. The Fréchet-Hoeffding limits (lower and upper) for the Bivariate Bernoulli Distribution *existence* are:

$$\text{Max}\left\{-\sqrt{\frac{CF_2\;CF_1}{(1-CF_2)\,(1-CF_1)}}\;,\;-\sqrt{\frac{(1-CF_2)\,(1-CF_1)}{CF_2\;CF_1}}\right\} \le \rho \le \sqrt{\frac{\text{Min}\{CF_2\,,\,CF_1\}\,(1-\text{Max}\{CF_2\,,\,CF_1\})}{\text{Max}\{CF_2\,,\,CF_1\}\,(1-\text{Min}\{CF_2\,,\,CF_1\})}}$$

Synergy Model for Development Option

- The optimal development investment depends on B, the reserve volume (larger B requires more wells, processing capacity, etc.)
  - Consider a linear function: $I_D = k_f + k_v\, B$, where k_f and k_v are parameters that depends on the oilfield location (water-depth).
- Synergy between two real options means that the joint real option value is higher than the sum of individual RO values.
  - Here this effect appear mainly for the development investment, because neighboring oilfields can share infrastructure, lowering the option exercise price (investment) for the joint development.
  - For the joint development, define the *synergy factor* $\gamma_{syn} \in [0, 1]$ as:

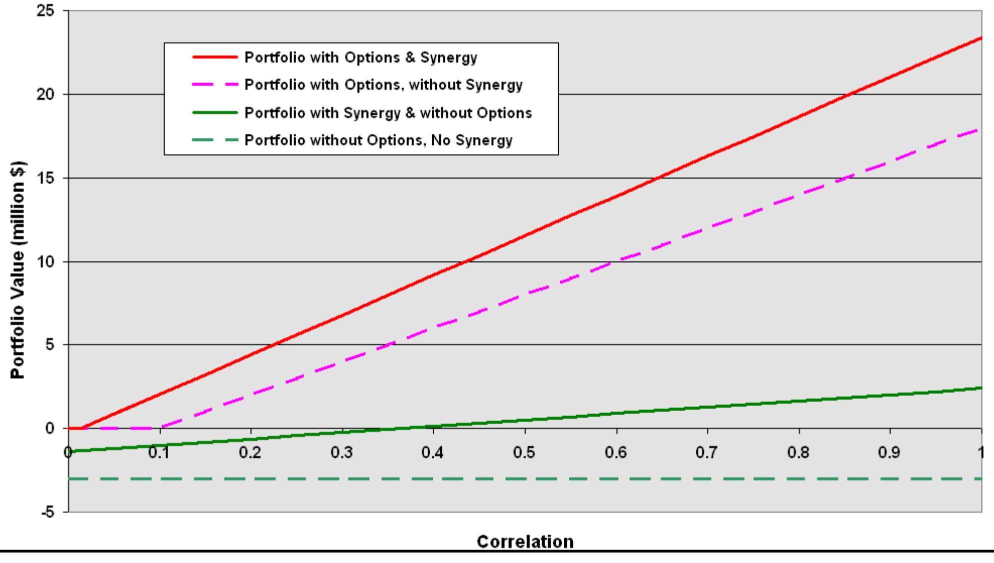
$$I_{D1+2} = I_{D1} + I_{D2} - \gamma_{syn}\,[I_{D1} + I_{D2} - (k_f + k_v\,(B_1 + B_2))]$$

- Synergy occurs only in case of *double exploratory success*, which occurs with probability that increases *linearly* with ρ:

$$prob_{syn} = \rho\,\sqrt{CF_1\,(1 - CF_1)\,CF_2\,(1 - CF_2)} + CF_1\,CF_2$$

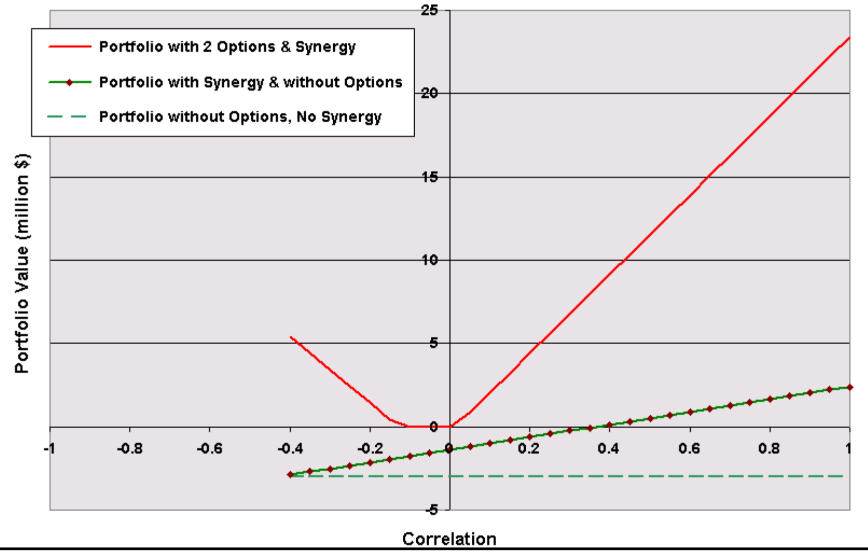
Expiring Portfolio with Learning and Synergy

- Portfolio is expiring (no option to delay). Compare the cases with and without option to drill the exploratory prospect, with learning (useful only with option) and with vs. without synergy.



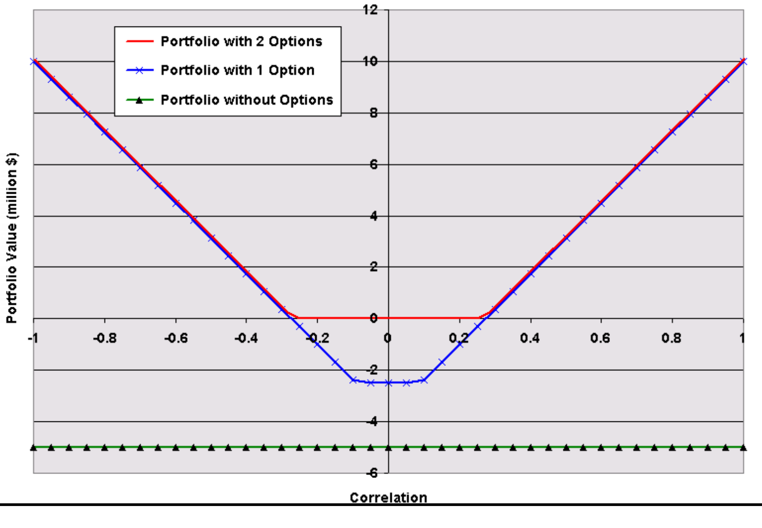
Expiring Portfolio: Learning with $\rho > 0$ and $\rho < 0$

- **If is allowed $\rho < 0$, we have also learning effect increasing value. But the Fréchet-Hoeffding limits don't allow learning in all range of ρ. For $CF_1 = CF_2$, it's possible $\rho = +1$, but not $\rho = -1$.**



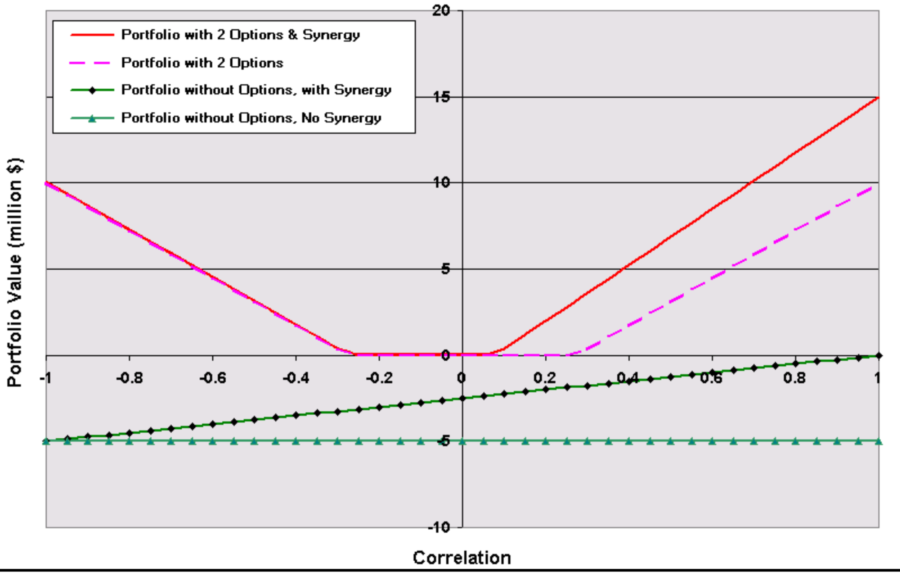
Expiring Portfolio: Learning in the Full Range of ρ

- **The only case that learning is possible for the whole range of ρ is when the marginal Bernoulli CF_1 and CF_2 are simultaneously *exchangeable* ($CF_1 = CF_2$) and *complementary* ($CF_1 = 1 - CF_2$).**
  - **It occurs only if $CF_1 = CF_2 = 50\ \%$. The chart focus only learning:**



Expiring Portfolio: Learning in the Full Range of ρ

- The same case ($CF_1 = CF_2 = 50\ \%$), but also showing the synergy effect.



The Option to Delay Development

- Let the development option (conditional to exploratory success) be $R_i(P, t)$, where i = 1 or 2 or 1 + 2 (joint development).
  - The partial differential equation and its boundary conditions are:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 R}{\partial P^2} + (r - \delta) P \frac{\partial R}{\partial P} - r R + \frac{\partial R}{\partial t} = 0$$

For R_1 or R_2 (non-joint development)

If $P = 0$, $R(0, t) = 0$

If $t = T$, $R(P, T) = \max[NPV(P), 0] = \max[q\, B\, P - I_D, 0]$

If $P = P^*$, $R(P^*, t) = NPV(P^*) = q\, B\, P^* - I_D$

If $P = P^*$, $\frac{\partial R(P^*, t)}{\partial P} = q\, B$

For R_{1+2} (joint development)

If $P = 0$, $R(0, t) = 0$

If $t = T$, $R_{1+2}(P, T) = \max(q_{1+2}\,(B_1 + B_2)\,P - I_{D1+2}, 0) = \max(NPV_{1+2}(P, T), 0)$

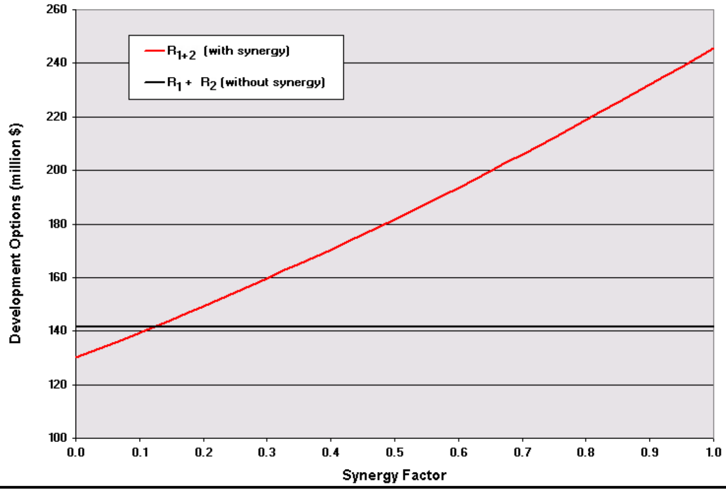
If $P = P^*_{1+2}$, $R_{1+2}(P^*_{1+2}, t) = q_{1+2}\,(B_1 + B_2)\,P^*_{1+2} - I_{D1+2} = NPV_{1+2}(P^*_{1+2}, t)$

If $P = P^*_{1+2}$, $\frac{\partial R_{1+2}(P^*_{1+2}, t)}{\partial P} = q_{1+2}\,(B_1 + B_2)$

  - If the oilfields are symmetric (same payoff value), then always we have $R_{1+2} \geq R_1 + R_2$. Hence, we wait for $P \geq P^*_{1+2}$ to exercise.
  - However, if the oilfields are sufficiently asymmetric, can occur $R_{1+2} < R_1 + R_2$. In this case we can exercise non-joint development

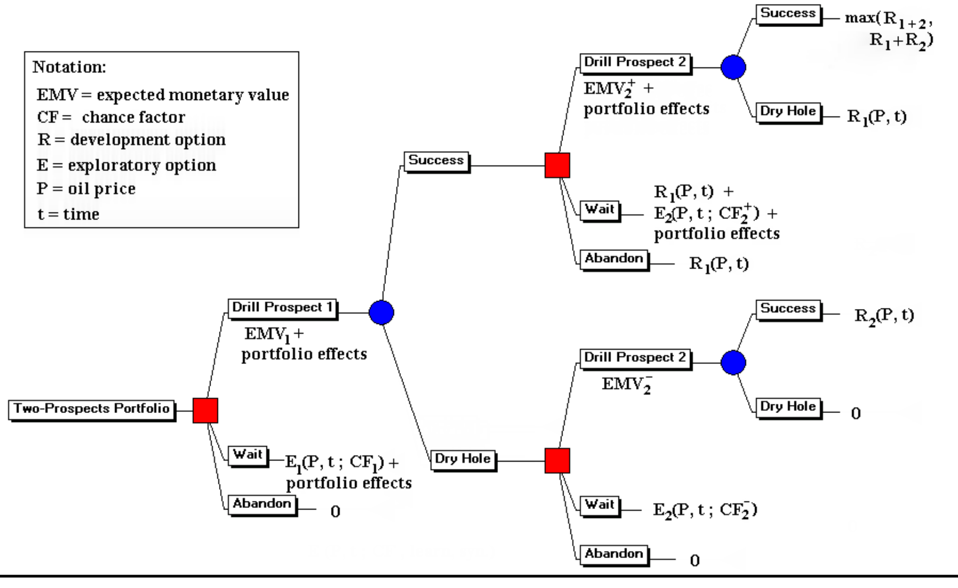
Asymmetric Oilfields Development Options

- **In this case, for low values of synergy factor (γ_{syn}), it is possible cases where $R_{1+2} < R_1 + R_2$, that is, instead wait for the joint development (exploiting synergy), we can wait for the best oilfield development threshold.**



Option to Delay in the Exploratory Portfolio

- **The decision tree shows the two-compound options portfolio with the option to delay, in addition to learning and synergy.**



PDE for Exploratory Options E_1 and E_2

- **The partial differential equations (PDE) for exploratory options E(P, t; CF) are the same, but boundary conditions are specific.**

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 E}{\partial P^2} + (r-\delta) P \frac{\partial E}{\partial P} - r E + \frac{\partial E}{\partial t} = 0$$

The case of E_2 (P, t; CF_2^-) is trivial because synergy effect is not possible anymore.

For E_2 (P, t; CF_2^+): If $P = 0$, $E_2(0, t) = 0$

If $t = T$,

$$E_2(P, T; CF_2^+) = \max\{0, -I_W + CF_2^+ \max[(NPV_{1+2} - \max(NPV_1, 0)), NPV_2]\}$$

If $P = P_2^{**}$, $R_{1+2} = NPV_{1+2}$ and $NPV_{1+2} > R_1 + R_2$,

$$E_2(P_2^{**}, t; CF_2^+) = -I_W + CF_2^+ (NPV_{1+2} - R_1)$$

If $P = P_2^{**}$, $R_2 = NPV_2$ and $NPV_2 > R_{1+2} - R_1$,

$$E_2(P_2^{**}, t; CF_2^+) = -I_W + CF_2^+ NPV_2$$

If $P = P_2^{**}$, $R_{1+2} = NPV_{1+2}$ and $NPV_{1+2} > R_1 + R_2$,

$$\frac{\partial E_2(P_2^{**}, t; CF_2^+)}{\partial P} = CF_2^+ \left[q_{1+2} (B_1 + B_2) - \frac{\partial R_1(P_2^{**}, t)}{\partial P}\right]$$

If $P = P_2^{**}$, $R_2 = NPV_2$ and $NPV_2 > R_{1+2} - R_1$,

$$\frac{\partial E_2(P_2^{**}, t; CF_2^+)}{\partial P} = CF_2^+ q_2 B_2$$

Boundary Conditions for Exploratory Option E_1

- **The first exploratory option to be exercised is more complex, because must consider both learning and synergy effect gains.**

For E_1(P, t; CF_1): If $P = 0$, $E_1(0, t) = 0$

If $t = T$,

$$E_1(P, T; CF_1) = \max[0, -I_W + CF_1 NPV_1 + CF_1 E_2(P, T; CF_2^+) + (1 - CF_1) E_2(P, T; CF_2^-) - E_2(P, T; CF_2)]$$

Where: $E_2(P, T; CF_2^+) = \max\{0, -I_W + CF_2^+ \max[(NPV_{1+2} - \max(NPV_1, 0)), NPV_2]\}$;

$E_2(P, T; CF_2^-) = \max\{0, -I_W + CF_2^- NPV_2\}$ and $E_2(P, T; CF_2) = \max\{0, -I_W + CF_2 NPV_2\}$

If $P = P_1^{**}$, $R_{1+2} = NPV_{1+2}$ and $NPV_{1+2} > R_1 + R_2$,

$$E_1(P_1^{**}, t; CF_1) = -I_W + CF_1 [-I_W + CF_2^+ NPV_{1+2} + (1 - CF_2^+) R_1] + (1 - CF_1) E_2(P_1^{**}, t; CF_2^-) - E_2(P_1^{**}, t; CF_2)$$

If $P = P_1^{**}$, $R_1 = NPV_1$ and $NPV_1 > R_{1+2} - R_2$,

$$E_1(P_1^{**}, t; CF_1) = -I_W + CF_1 NPV_1 + CF_1 E_2(P_1^{**}, t; CF_2^+) + (1 - CF_1) E_2(P_1^{**}, t; CF_2^-) - E_2(P_1^{**}, t; CF_2)$$

If $P = P_1^{**}$, $R_{1+2} = NPV_{1+2}$ and $NPV_{1+2} > R_1 + R_2$,

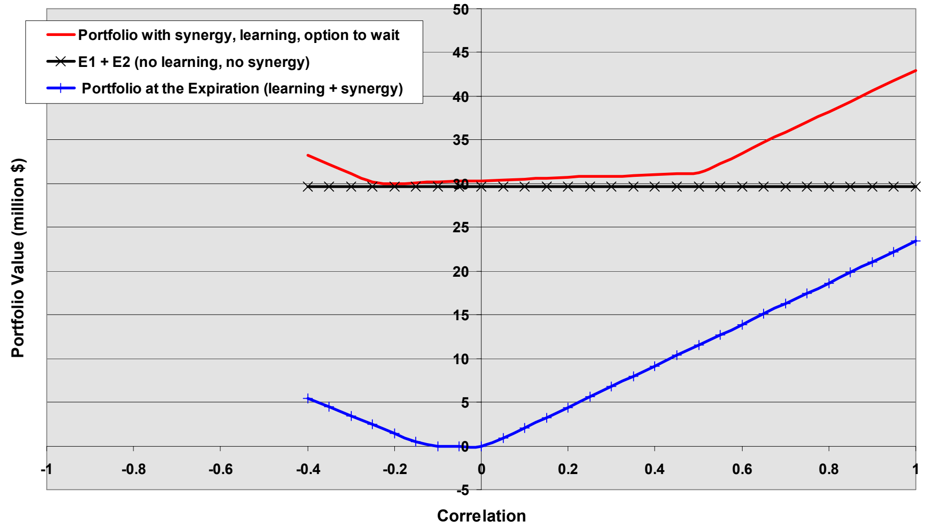
$$\frac{\partial E_1(P_1^{**}, t; CF_1)}{\partial P} = CF_1 \left[CF_2^+ q_{1+2} (B_1 + B_2) + (1 - CF_2^+) \frac{\partial R_1(P_1^{**}, t)}{\partial P}\right] + (1 - CF_1) \frac{\partial E_2(P_1^{**}, t; CF_2^-)}{\partial P} - \frac{\partial E_2(P_1^{**}, t; CF_2)}{\partial P}$$

If $P = P_1^{**}$, $R_1 = NPV_1$ and $NPV_1 > R_{1+2} - R_2$,

$$\frac{\partial E_1(P_1^{**}, t; CF_1)}{\partial P} = CF_1 \left[q_1 B_1 + \frac{\partial E_2(P_1^{**}, t; CF_2^+)}{\partial P}\right] + (1 - CF_1) \frac{\partial E_2(P_1^{**}, t; CF_2^-)}{\partial P} - \frac{\partial E_2(P_1^{**}, t; CF_2)}{\partial P}$$

Portfolio Value with vs. without Option to Delay

- This chart compares the case with all effects (option to delay, learning and synergy) with the cases without learning & synergy and the case with all, but without option to delay.



Conclusions

- We analyze two-assets portfolio, each asset is a compound real-option, considering learning and synergy interactions.
  - The application is for petroleum exploration + development, but this theory can be adapted for R&D portfolio.
- Contrasting most literature on portfolio of real options, this paper shows that positive correlation (ρ) or the correlation-squared (ρ^2) can *increases* the portfolio value when considering synergy and learning effects.
  - We show that the *synergy* effect value increases with ρ because the probability of joint development increases with ρ.
  - We show that the *learning* effect value is (non-strictly) increasing with ρ^2 (not ρ itself). I show also the (F-H) limits for learning.
- We include also the *option to delay*, interacting with learning and synergy. Many other extensions can be considered.
- Thank you very much for your time!

APPENDIX

SUPPORT SLIDES

Synergy Model for Development Option

- **The joint development NPV equation is (synergy only at I_{D1+2}):**

$$NPV_{1+2} = (q_1 B_1 + q_2 B_2) P - I_{D1+2}$$

- **Proposition 1: Consider the two exploratory prospects portfolio with chance factors given by correlated Bernoulli distributions with correlation coefficient ρ. Then:**
  - **The learning gain from the first exploratory option exercise is (non-strictly) increasing with ρ^2.**
  - **The expected synergy gain with double exploratory option exercise is (non-strictly) increasing with the correlation ρ.**
- **For the now-or-never case, the portfolio value with learning and synergy is given by:**

$$\Pi_{1+2} = \max\{0, -I_W + CF_1 \max[NPV_1, -I_W + CF_2^{+} NPV_{1+2} + (1 - CF_2^{+}) NPV_1] + $$
$$+ (1 - CF_1) \max[0, -I_W + CF_2^{-} NPV_2]\}$$